1. Compute the following limits.

(a) \[
\lim_{{x \to 3}} \frac{x^3 - 27}{x^3 + x - 6}
\]
\[
= \frac{3^3 - 27}{3^3 + 3 - 6} = \frac{27 - 27}{27 + 3 - 6} = \frac{0}{6} = 0
\]

Not an indeterminate form. Thus, \(1\)

(b) \[
\lim_{{x \to \pi}} \frac{\sin x}{\pi^2 - x^2}
\]
\[
= \sin \pi = 0, \text{ so this is of the form } \frac{0}{0} \text{. Applying L' Hospitals Rule (1)}
\]
\[
\lim_{{x \to \pi}} \frac{\sin x}{\pi^2 - x^2} = \lim_{{x \to \pi}} \frac{(\sin x)'}{(\pi^2 - x^2)'} = \lim_{{x \to \pi}} \frac{\cos x}{-2x}
\]
\[
= \frac{\cos \pi}{-2\pi} = \frac{-1}{-2\pi} = \frac{1}{2\pi}
\]
2. A farmer has 170 feet of fencing with which to enclose a pen for his sheep. He wants to build the pen adjacent to a 30 ft long barn, and will not need to put fencing against the barn. What is the area of the largest pen he can build?

Trying to maximize \[ \text{Area} = xy \] (1)

Under the constraint

\[
\begin{align*}
    x - 30 + y + x + y &= 170 \\
    2x + 2y - 30 &= 170 \\
    2x + 2y &= 200 \\
    x + y &= 100
\end{align*}
\] (1)

So \( y = 100 - x \). Plugging this into area gives

\[ \text{Area} = A(x) = x(100 - x) = 100x - x^2 \]

Interval: \( x \geq 30 \) \( \Rightarrow \) \( 30 \leq x \leq 100 \) (1)

Optimize \[ A'(x) = 100 - 2x = 0 \Rightarrow x = 50 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( A(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30(100-30) = 30 \times 70 = 2100</td>
</tr>
<tr>
<td>50</td>
<td>50(100-50) = 50 \times 50 = 2500</td>
</tr>
<tr>
<td>100</td>
<td>100(100-100) = 0</td>
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</tbody>
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So the maximum area is \[ 2500 \text{ ft}^2 \] (1)