1. Find the vertical and horizontal asymptotes of the following functions:

\[
\lim_{x \to \infty} \frac{e^{3x} + 4}{e^{2x} - 8} = \frac{\infty}{\infty} \\
\text{L'Hopital's Rule:} \quad \lim_{x \to \infty} \frac{3e^{3x}}{2e^{2x}} = \frac{\infty}{\infty} \\
= \lim_{x \to \infty} \frac{3}{2} e^{x} = \infty \\
\text{vertical asymptote:} \quad y = -\frac{1}{2}
\]

\[
f(x) = \frac{e^{3x} + 4}{e^{2x} - 8} \\
\lim_{x \to \infty} \frac{e^{3x} + 4}{e^{2x} - 8} = \frac{-\infty}{0} \\
\text{vertical asymptote:} \quad x = \frac{1}{2} \ln 8
\]

\[
g(x) = \frac{e^{2x} - 5}{e^{2x} + 2} \\
\lim_{x \to \infty} \frac{e^{2x} - 5}{e^{2x} + 2} = \frac{\infty}{\infty} \\
\text{L'Hopital's Rule:} \quad \lim_{x \to \infty} \frac{2e^{2x}}{2e^{2x}} = \frac{\infty}{\infty} \\
= \lim_{x \to \infty} \frac{1}{1} = 1 \\
\text{horizontal asymptote:} \quad y = 1
\]

\[
g(x) = \frac{e^{2x} - 5}{e^{2x} + 2} \\
\lim_{x \to \infty} \frac{e^{2x} - 5}{e^{2x} + 2} = \frac{0 - 5}{0 + 2} = \frac{-5}{2} \\
\text{horizontal asymptote:} \quad y = -\frac{5}{2}
\]

\[
\text{Never true!} \\
\text{No vertical asymptotes.}
\]
2.

(a) Use implicit differentiation to find \( \frac{dy}{dx} \) if \( x \) and \( y \) satisfy the equation

\[
\frac{d}{dx} \left( x^2y + 3y^2 = -2x^3 + 4y + 8 \right) = 2x \cdot y + x^2 \frac{dy}{dx} + 6y \frac{dy}{dx} = -4x + 4 \frac{dy}{dx}
\]

\[
(x^2 + 6y - 4) \frac{dy}{dx} = 4x - 2xy
\]

\[
\frac{dy}{dx} = \frac{-4x - 2xy}{x^2 + 6y - 4} = \frac{4x + 2xy}{4 - x^2 - 6y}
\]

(b) Use part (a) to write the equation for the tangent line to the graph of the equation above at the point (1,2)

\[
y - y_1 = m(x - x_1)
\]

\[
m = \frac{dy}{dx} \bigg|_{(1,2)} = \frac{4(1) + 2(1)(2)}{4 - (1)^2 - 6(2)} = \frac{4 + 4}{4 - 1 - 12} = \frac{8}{-9}
\]

Then

\[
y - 2 = \frac{-8}{9} (x - 1)
\]

\[
y = \frac{-8}{9} x + \frac{8}{9}
\]

or

\[
y = \frac{-8}{9} x + \frac{26}{9}
\]