1. You are producing widgets to sell for a profit. If the price of the widgets is $p$ dollars, then you believe you will be able to sell $x = 300 - 2p$ widgets. The cost to produce $x$ widgets is given by

$$C(x) = 20 + 30x - 5x^2 + x^3.$$  

(a) How many widgets should you produce to maximize your profit?

(b) How much are you selling the widgets for?

(c) What is the average cost to produce the widgets?

**Rearranging:**

$$p(x) = \frac{300 - x}{2} = 150 - \frac{x}{2} \quad (1)$$

**Revenue:**

$$R(x) = xp(x) = 150x - \frac{x^2}{2}.$$

**Profit:**

$$P(x) = R(x) - C(x) = 150x - \frac{x^2}{2} - 20 - 30x + 5x^2 - x^3.$$  

To maximize profit, set $P'(x) = 0$.

$$P'(x) = 150 - x - 30 + 10x - 3x^2 \quad (1)$$

$$= 120 + 9x - 3x^2 = 0$$

$$\Rightarrow x^3 - 3x - 40 = 0 = (x-8)(x+5).$$

So, you should produce **8 widgets to maximize profit**.

(b) $$P(8) = \frac{\$8}{2} = \$1680 \quad (1)$$

(c) **Average Cost**

$$A(x) = \frac{C(x)}{x} = \frac{20}{x} + 30 - 5x + x^2$$

$$A(8) = \frac{20}{8} + 30 - 40 + 64 = 54 + \frac{20}{8} = \frac{560}{8} = \$70$$
2. Compute the indefinite integral

\[ \int \cos(x) + 4e^x + x^3 \, dx \]

\[ = \sin(x) + 4e^x + \frac{1}{4}x^4 + C. \]

3. Approximate the area under the curve \( y = x^2 + 2x \) between \( x = 1 \) and \( x = 3 \) using a right-endpoint Riemann Sum, with \( n = 2 \).

\[ \Delta x = \frac{b-a}{n} = \frac{3-1}{2} = 1 \]

(1)

Height of first rectangle:

\( f(2) = 2^2 + 2 \cdot 2 = 8 \)

(1)

Second:

\( f(3) = 3^2 + 2 \cdot 3 = 15 \)

(1)

\[ A \approx \Delta x(f(1) + f(3)) \]

\[ = 1(8 + 15) = 23 \]

(1)