1. Use differentials or linear approximations to approximate the value of \( \sqrt{16.1} \).

\[
\frac{d}{dx} x = \frac{x}{2}, \quad \frac{d}{dx} x = \frac{1}{2} x^{-1/2}.
\]

\[
f(16.1) \approx f(16) + f'(16)(\Delta x) \approx 4 + \frac{1}{2}(1)(0.1) = 4.0125.
\]

2. The Kinetic Energy of an object [in Joules] is given by the formula

\[
KE(v) = \frac{1}{2}mv^2
\]

where \( v \) is the velocity of the ball [in m/s] and \( m \) is the mass [in kg]. Someone throws a 1 kg medicine ball at the wall and you measure the speed of it as 10 m/s. [The mass is exactly 1 kg, so there is no error in that measurement.] If your measurement of the speed is within .5 m/s, what is the approximate propagated error \( \Delta KE \) in your calculation of the Kinetic Energy of the ball?

\[
\Delta KE \approx dKE = KE'(v_0) \Delta v.
\]

\[
KE'(v) = \frac{d}{dv}\left(\frac{1}{2}mv^2\right) = mv
\]

So

\[
KE'(v_0) = (1)(10) = 10 \text{ kg m/s}
\]

Thus

\[
\Delta KE \approx (10 \text{ kg m/s})(0.5 \text{ m/s}) = \pm 5 \text{ kg m/s}^2
\]
3. Let \( f(x) = 2x^3 - 9x^2 = x^2(2x - 9) \).

(a) Find all “critical numbers” or “critical points” of \( f \).

(b) Find the absolute maximum and minimum values of \( f \) on \([-1, 5]\).

(c) What does the Mean Value Theorem say about some point \( c \) between \(-1 \text{ and } 5\)?

(a) \( f'(x) = 6x^2 - 18x = 6x(x - 3) \) \( (1) \)

Critical numbers: \( f'(x) = 0 \) or \( f'(x) \) does not exist.

\( f'(x) \) exists everywhere

\( f'(x) = 0 \Rightarrow 6x(x-3) = 0 \Rightarrow x = 0, x = 3 \) \( (1) \)

(b) Need to check critical numbers and endpoints.

\( f(-1) = (-1)^2(2(-1)-9) = 1 \cdot (-2 - 9) = -11 \) \( (1) \)

\( f(0) = 0^2 (2(0)-9) = 0 \)

\( f(3) = (3)^2(2(3)-9) = 9 \cdot (6 - 9) = -27 \leq \text{Minimum} \)

\( f(5) = (5)^2(2(5)-9) = 25 \cdot 1 = 25 \leq \text{Maximum} \) \( (1) \)

So Absolute min is \(-27\) at \( x = 3 \)

Absolute max is \( 25 \) at \( x = 5 \).

(c) MVT says that there is a \( c \) between \(-1 \text{ and } 5\) with

\[ b - c = \frac{f(5) - f(-1)}{f'(c)} = \frac{25 - (-11)}{6} = \frac{36}{6} = 6. \] \( (1) \)