1. Let $\mathbf{F} = (yz, xz, xy)$. Compute the integral

$$\int_C \mathbf{F} \cdot d\mathbf{s}$$

for the curve $C(t) = (3t^2 + \cos(\pi t^2), 2t^4 + t^3 - t + 1)$ for $t = 0$ to $1$. Hint: Is $\mathbf{F}$ conservative?

$\mathbf{F}$ is conservative with $\mathbf{F} = \nabla (xyz)$, thus

$$\int_C \mathbf{F} \cdot d\mathbf{s} = \phi(C(1)) - \phi(C(0))$$

$$= \phi(3,1,2) - \phi(1,1,1)$$

$$= \phi(2,1,1) - \phi(1,1,1) = 8 - 1 = \boxed{7}$$

2. Determine whether or not the following vector fields are conservative. If they are conservative, find a potential function.

(a) $\mathbf{F} = (y^3, x^2, \sin(z))$.

(b) $\mathbf{G} = (3x^2 + \sin(z), 2yz, y^2 + x\cos(z))$.

(a) No. $\frac{\partial F}{\partial y} = 2y \neq \frac{\partial F}{\partial x}$

(b) Yes. $\frac{\partial \phi}{\partial x} = 3x^2 + \sin z \rightarrow \phi = x^3 + x\sin z$

$$\frac{\partial \phi}{\partial y} = 2yz \rightarrow \phi = y^2z$$

$$\frac{\partial \phi}{\partial z} = y^2 + x\cos z \rightarrow \phi = y^2z + x\sin z$$

Thus,

$$\phi = x^3 + y^2z + x\sin z$$
3. Let \( f(x, y, z) = 9z + 2x \) and let \( C \) be the curve \( c(t) = (t, t^2, t^3) \) for \( t = 0 \) to \( 1 \). Compute

\[
\int_{C} f(x, y, z) \, ds.
\]

\[
f(c(t)) = 9t^3 + 2t.
\]

\[
c'(t) = \langle 1, 2t, 3t^2 \rangle.
\]

\[
||c'(t)|| = \sqrt{1 + 4t^2 + 9t^4}
\]

\[
\int_{C} f(x, y, z) \, ds = \int_{0}^{1} f(c(t)) \, ||c'(t)|| \, dt
\]

\[
= \int_{0}^{1} (9t^3 + 2t) \sqrt{1 + 4t^2 + 9t^4} \, dt
\]

\[
= \frac{1}{4} \int \sqrt{u} \, du
\]

\[
= \frac{2}{3} \cdot \frac{1}{4} u^{3/2} = \frac{1}{6} \left( 1 + 4t^2 + 9t^4 \right)^{3/2}
\]

\[
= \frac{1}{6} \left( 1 + 4t^2 + 9t^4 \right)^{3/2} - 1
\]

\[
= \frac{1}{6} \left( 1 + 4t^2 + 9t^4 \right)^{3/2} - 1
\]

\[
\int_{0}^{1} \left( 1 + 4t^2 + 9t^4 \right)^{3/2} \, dt
\]