Math 244: Differential Equations for Engineers

Summer 2003, Section C1

Assignment 3: Modeling with Second-Order Equations (13 points)

Due Monday, June 16, 2003

In this assignment, you will be asked to use second-order differential equations to work through a relatively complicated, multi-step modeling process. Please write out your solutions to the problems below on one or more separate sheets of paper. Write neatly and in a well-organized fashion. Write in clear, complete sentences, using diagrams and equations where appropriate. Show all your work, including the methods you use for solving the differential equations you are asked to solve; you will not receive full credit for simply writing down answers.

1. A mass weighing 6 lb stretches a spring 4 in. Suppose the mass is pulled downward an additional 3 in and then released. Assume first that there is no damping, and no external force acts on the mass.

a. (2 points) Determine the position u of the mass at any time t.

b. (2 points) Find the frequency, period, amplitude, and phase of the motion.

2. Now suppose the mass is damped by attaching it to a viscous damper. Suppose the damper exerts a force of a lb when the velocity of the mass is 5 ft/sec.

a. (2 points) If $a = 1$, find the position of the mass u at any time t.

b. (3 points) Find the value of a such that the system is critically damped.

3. Finally, suppose the mass is undamped, but is acted on by an external force of $3\cos\omega t$ lb.

a. (2 points) If $\omega = 2$, find the position of the mass u at any time t.

b. (2 points) Find the value of ω such that resonance occurs.