Chvátal's theorem on Hamiltonian degree sequences

Let $(d_1, ..., d_n)$ be a sequence of nonnegative integers with $d_1 \leq ... \leq d_n$. Suppose a graph G has degree sequence $(a_1, ..., a_n)$. We say G's degree sequence is *pointwise greater* than $(d_1, ..., d_n)$ if $a_i \geq d_i$ for every i, $1 \leq i \leq n$.

Definition: The sequence $(d_1, ..., d_n)$ is Hamiltonian if every graph G whose degree sequence is pointwise greater than $(d_1, ..., d_n)$ contains a Hamiltonian cycle.

Examples: Dirac's theorem says that the sequence $(\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil, ..., \lceil \frac{n}{2} \rceil)$ is Hamiltonian. The sequence $(2, 2, 2, 2, 2, 2)$ is not Hamiltonian, even though the 6-cycle has that degree sequence, because there are other graphs (e.g. the graph consisting of two disjoint triangles) which have that degree sequence and have no Hamiltonian cycle.

Theorem (Chvátal, 1972): The sequence $(d_1, ..., d_n)$ is Hamiltonian if and only if, for every $i < n/2$, if $d_i \leq i$ then $d_{n-i} \geq n - i$ (or, to put it another way: if either $d_i > i$ or $d_{n-i} \geq n - i$ for every such i).

Example: The sequence $(3, 3, 3, 9, 9, 9, 9, 9, 9, 9, 9, 9)$ satisfies Chvátal's condition and is therefore Hamiltonian. Note that there are graphs with this degree sequence that do not satisfy the hypotheses of Dirac's theorem or Ore's theorem.