

22 June 2007

Quiz 9: Math 135, Section C7

1. If $5xy = 10$ and $\frac{dx}{dt} = -2$, then find $\frac{dy}{dt}$ when $x = 1$.

Right now, a truck is exactly 250 miles east of a car. The truck is pointed west, and the car is pointed north. They proceed in the direction they are pointed, with the truck traveling 25mph and the car traveling 50mph.

2. Draw a picture of this scenario.

3. Let D denote the distance between the car and the truck. Give an equation involving D and two other variables **but not** t .

4. Using related rates and the equation from the previous question, find $\frac{dD}{dt}$ when $t = 2$.

5. (**extra credit - 2pt**) Find the time t where $\frac{dD}{dt} = 0$. What does this mean in terms of D ?

1. Since $5xy = 10$ then it means $xy = 2$, so differentiating with respect to t gives

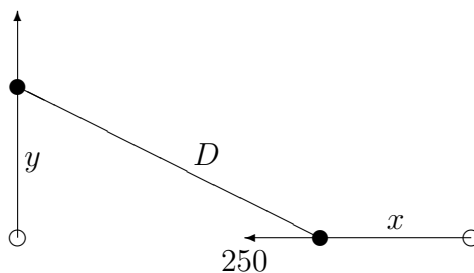
$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0$$

Since we know that $xy = 2$, then it means that when $x = 1$ then $y = 2$, so plugging in we get

$$(1) \frac{dy}{dt} + (2)(-1) = 0$$

So solving, we get $\frac{dy}{dt} = 2$.

2. The picture is as follows:



3. Our equation would be

$$(250 - x)^2 + y^2 = D^2$$

4. Differentiating the previous equation with respect to t we get

$$2(250 - x) - \frac{dx}{dt} + 2y \frac{dy}{dt} = 2D \frac{dD}{dt}$$

When $t = 2$, then x is 50 and y is 100. $\frac{dx}{dt} = 25$ and $\frac{dy}{dt} = 50$. At this point, $D = \sqrt{150^2 + 100^2}$ so plugging in we get

$$-2(150)(25) + 2(100)(50) = 2\sqrt{150^2 + 100^2} \frac{dD}{dt}$$

or

$$\frac{dD}{dt} = \frac{-2(150)(25) + 2(100)(50)}{2\sqrt{150^2 + 100^2}}$$