x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	−1	0

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	−1	0	1

$\sin(x+2\pi)=\sin(x)$
$\cos(x+2\pi)=\cos(x)$

Identities: $\sin^2 x+\cos^2 x=1$, $1+\tan^2 x=\sec^2 x$, $\sin(2x)=2\sin x\cos x$, $\cos(2x)=\cos^2 x-\sin^2 x$.

Addition: $\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$ $\quad\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$ $\quad\pi\approx 3.1416$.

Exponentials and logarithms: $a,b,t,u,y>0$, r,v,w,x any real numbers: $a^{v+w}=a^v a^w$, $a^{vw}=(a^v)^w$, $a^{-v}=1/a^v$, $a^0=1$, $(ab)^v=a^v b^v$, $\log_a(t)=\ln(t)/\ln(a)$. $e^x=y$ is equivalent to $x=\ln y$, $e^{\ln y}=y$, $\ln(e^x)=x$. $\ln(tu)=\ln(t)+\ln(u)$, $\ln(u^r)=r\ln(u)$, $\ln(1/u)=-\ln(u)$, $\ln(1)=0$, $e\approx 2.718$.

$f(x)$	$\int f(x)\,dx$
$x^r, r\neq -1$	$x^{r+1}/(r+1)+C$
x^{-1}	$\ln\lvert x\rvert+C$
e^x	e^x+C
$a^x, a\neq 1$	$a^x/(\ln a)+C$

$f(x)$	$\int f(x)\,dx$
$\sin x$	$-\cos x+C$
$\cos x$	$\sin x+C$
$\sec^2 x$	$\tan x+C$
$\sec x\tan x$	$\sec x+C$

$f(x)$	$\int f(x)\,dx$
$1/\sqrt{a^2-x^2}, a\neq 0$	$\sin^{-1}(x/a)+C$
$1/(a^2+x^2), a\neq 0$	$(1/a)\tan^{-1}(x/a)+C$
$\tan x$	$-\ln\lvert\cos x\rvert+C$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert+C$

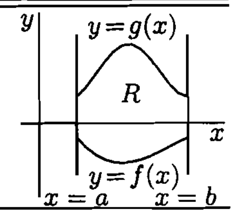
Areas and Volumes of Solids of Rotation, see figure at the right

R is the region bounded by $y=g(x)$, $y=f(x)$, $x=a$, $x=b$, with $g(x)\geq f(x)$ for x in $[a,b]$.

Area of R: $\int_a^b (g(x)-f(x))\,dx$

Volume of solid found by rotating R around the x-axis: $\int_a^b \pi((g(x))^2-(f(x))^2)\,dx$, $(f(x)\geq 0)$

Volume of solid found by rotating R around the y-axis: $\int_a^b 2\pi x(g(x)-f(x))\,dx$, $(a\geq 0)$



Average value of $y=f(x)$ on the interval $[a,b]$: $(1/(b-a))\int_a^b f(x)\,dx$

Integration by parts: $\int u\,dv=uv-\int v\,du$. Choose dv to be easy to integrate and so that $\int v\,du$ is simpler than $\int u\,dv$.

$\int \sin^m x\cos^n x\,dx$ Reduce to a sum of integrals of the type: $\int \sin^j x(\cos x\,dx)$ and $\int \cos^k x(\sin x\,dx)$

m odd: Group $\sin x$ with dx. Replace $\sin^{m-1}x$ using $\sin^2 x=1-\cos^2 x$. Let $u=\cos x$, $du=-\sin x\,dx$. Expand.

n odd: Group $\cos x$ with dx. Replace $\cos^{n-1}x$ using $\cos^2 x=1-\sin^2 x$. Let $u=\sin x$, $du=\cos x\,dx$. Expand.

m, n both even: Use $\sin^2 x=(1-\cos(2x))/2$, $\cos^2 x=(1+\cos(2x))/2$. Possibly repeat, or use an earlier case.

$\int \sec^m x\tan^n x\,dx$ Reduce to a sum of integrals of the type: $\int \sec^j x(\sec x\tan x\,dx)$ and $\int \tan^k x(\sec^2 x\,dx)$

m even: Group $\sec^2 x$ with dx. Replace $\sec^{m-2}x$ using $\sec^2 x=1+\tan^2 x$. Let $u=\tan x$, $du=\sec^2 x\,dx$. Expand.

n odd: Group $\sec x\tan x$ with dx. Replace $\tan^{n-1}x$ using $\tan^2 x=\sec^2 x-1$. Let $u=\sec x$, $du=\sec x\tan x\,dx$.

m odd, n even: Use $\tan^2 x=\sec^2 x-1$ to express as a sum of integrals of $\sec^m x$, m odd. Use integration by parts.

Integrals involving $\sqrt{a^2-x^2}$. Set $x=a\sin\theta$. Then, $dx=a\cos\theta\,d\theta$, $\sqrt{a^2-x^2}=a\cos\theta$.

Integrals involving $\sqrt{a^2+x^2}$. Set $x=a\tan\theta$. Then, $dx=a\sec^2\theta\,d\theta$, $\sqrt{a^2+x^2}=a\sec\theta$.

Integrals of rational functions: $\int (f(x)/g(x))\,dx$, with $f(x), g(x)$ polynomials.

(1) If $\deg f(x)\geq\deg g(x)$, divide $g(x)$ into $f(x)$ using long division of polynomials to get quotient $q(x)$ and remainder $r(x)$ with $\deg r(x)<\deg g(x)$. Then $\int (f(x)/g(x))\,dx=\int q(x)\,dx+\int (r(x)/g(x))\,dx$. Use next method on last integral.

(2) If $\deg f(x)<\deg g(x)$, use the method of partial fractions. First factor $g(x)$ into a product of linear factors $x+A$ and quadratic factors x^2+Bx+C with no real roots, or $g(x)=\underbrace{(x+A)^m\cdots\cdots}_{\text{linear factors}}\underbrace{(x^2+Bx+C)^m\cdots\cdots}_{\text{quadratic factors}}$.

Then, $f(x)/g(x)$ is a sum of terms:

$$\underbrace{\frac{D_1}{(x+A)}+\frac{D_2}{(x+A)^2}+\cdots+\frac{D_m}{(x+A)^m}}_{m\text{ terms for each linear factor }(x+A)^m}\quad\text{and}\quad\underbrace{\frac{E_1x+F_1}{(x^2+Bx+C)}+\frac{E_2x+F_2}{(x^2+Bx+C)^2}+\cdots+\frac{E_nx+F_n}{(x^2+Bx+C)^n}}_{n\text{ terms for each quadratic factor }(x^2+Bx+C)^n}$$

To find the constants D_i, E_j, F_k, multiply both sides by $g(x)$. Then, multiply the terms out and equate the coefficients of the same powers of x on each side of the equation. Finally, solve the resulting system of linear equations.

Suppose the integral $\int_a^b f(x)\,dx$ is approximated by dividing $[a,b]$ into n equal segments. Let K be an upper bound for $|f''(x)|$ on $[a,b]$. The midpoint rule error is at most $K(b-a)^3/24n^2$, the trapezoidal error at most $K(b-a)^3/12n^2$.

$\int_a^\infty f(x)\,dx=\lim_{t\to\infty}\int_a^t f(x)\,dx$ $\quad\int_{-\infty}^b f(x)\,dx=\lim_{t\to-\infty}\int_t^b f(x)\,dx$ $\quad\int_1^\infty \frac{dx}{x^a}$ converges if $a>1$, diverges if $a\leq 1$.

Compar: If $0\leq f(x)\leq g(x)$, let $I_f=\int_a^\infty f(x)\,dx$, $I_g=\int_a^\infty g(x)\,dx$. If I_g converges, so does I_f. If I_f diverges, so does I_g.

L'Hospital: If $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$.

Some limits: If $a>0$, $\lim_{x\to\infty}x^a=\infty$, $\quad\lim_{x\to\infty}1/x^a=0$. $\quad$ If $a>1$, $\lim_{x\to\infty}a^x=\infty$, $\lim_{x\to\infty}1/a^x=0$.

$\lim_{x\to\infty}\ln x=\infty$. $\quad\lim_{x\to\pm\infty}\tan^{-1}x=\pm\pi/2$.

Arc length of $y=f(x)$ from $x=a$ to $x=b$ is $\int_a^b\sqrt{1+(dy/dx)^2}\,dx=\int_a^b\sqrt{1+(f'(x))^2}\,dx$.

Surface area of surface found by rotating $y=f(x)$ from $x=a$ to $x=b$ around the x-axis is $\int_a^b 2\pi y\sqrt{1+(dy/dx)^2}\,dx=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$.

Differential equations–separation of variables. Factor one side of the equation so that $dy/dx = f(x)g(y)$. Then $dy/g(y) = f(x)dx$. Integrate, not forgetting the constant C of integration. Use the initial condition $(x, y) = (x_0, y_0)$ to find C.

Differential Equation $dy/dt = k(y - b)$ has solution $y = b + Ce^{kt}$, with C an arbitrary constant. It can be used to solve problems involving Newton's Law of Cooling: $y' = -k(y - T_0)$, where T_0 is the temperature of a medium in which an object of temperature $y(t)$ has been placed.

$\lim_{n\to\infty} \frac{n^a}{b^n} = 0$, if $b > 1$; $\lim_{n\to\infty} \frac{a^n}{n!} = 0$; f, g poly., $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{an^j + \text{lower terms}}{bn^k + \text{lower terms}} = \begin{cases} a/b, & \text{if } j = k; \\ 0, & \text{if } j < k; \pm\infty, \text{ if } j > k. \end{cases}$

Geometric series: $\sum_{n=0}^{\infty} x^n = 1/(1 - x)$, if $|x| < 1$. p-series: $\sum_{n=1}^{\infty} 1/n^p$ converges if $p > 1$, diverges if $p \leq 1$.

Divergence Test: The series $\sum a_n$ diverges if either $\lim_{n\to\infty} a_n$ does not exist or exists and is not 0.

Integral Test: Suppose that f is a positive, continuous, decreasing function. Let $a_n = f(n)$ for each positive integer n. Then, the integral $\int_1^{\infty} f(x)dx$ and the series $\sum_{n=1}^{\infty} a_n$ converge or diverge together. Use for $\sum 1/n^p$, $\sum 1/(n(\ln n)^p)$.

Comparison Test: Assume $0 \leq a_n \leq b_n$. If $\sum b_n$ converges, so does $\sum a_n$. If $\sum a_n$ diverges, so does $\sum b_n$.

Limit Comparison: If $a_n, b_n > 0$, $\lim_{n\to\infty}(a_n/b_n) = L$, with $L \neq 0, \infty$, $\sum a_n$ and $\sum b_n$ both converge or diverge.

Alternating Series: If $\sum_{n=0}^{\infty} (-1)^n a_n$ is a series with $a_n \geq 0$, $a_{n+1} \leq a_n$ for all n and $\lim_{n\to\infty} a_n = 0$, the series converges.

Absolute Convergence: The series $\sum a_n$ converges absolutely if $\sum |a_n|$ converges. Then also $\sum a_n$ converges.

Conditional Convergence: The series $\sum a_n$ converges conditionally if $\sum a_n$ converges but $\sum |a_n|$ diverges.

Ratio Test: Suppose that $\lim_{n\to\infty} |a_{n+1}/a_n| = L$. If $L < 1$, $\sum a_n$ converges absolutely. If $L > 1$, $\sum a_n$ diverges. If $L = 1$, the test fails. Often useful for series involving a^n or $n!$. For power series, use this to find the radius of convergence. The Root Test is the same as the ratio test, but supposing instead that $\lim_{n\to\infty} |a_n|^{1/n} = L$.

Radius of convergence: For the power series, $\sum c_n(x - a)^n$, there is a number $R \geq 0$ so that if $|x - a| < R$, the series converges absolutely and if $|x - a| > R$, the series diverges. If $|x - a| = R$, the series may converge or diverge. Find R with the ratio test. The interval of convergence I is the set of points where the series converges. I contains the interval $(a - R, a + R)$ and may or may not contain the boundary points $a - R$ and $a + R$.

Taylor Polynomials: The n-th Taylor poly. of f is $T_n(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2! + \cdots + f^n(a)(x-a)^n/n!$, assuming f has n derivatives at a. Also $f(x) = T_n(x) + R_n(x)$, $|R_n(x)| \leq M|x - a|^{n+1}/(n+1)!$, with M an upper bound of $|f^{n+1}(t)|$ with t in the interval with endpoints a and x.

$e^x = \sum_{n=0}^{\infty} x^n/n!$ $\quad$ $\sin x = \sum_{n=0}^{\infty} (-1)^n x^{2n+1}/(2n+1)!$ $\quad$ $\cos x = \sum_{n=0}^{\infty} (-1)^n x^{2n}/(2n)!$

Parametric curves: If $x = f(t), y = g(t)$, then the slope of the tangent line is given by $\frac{dy}{dx} = (\frac{dy}{dt})/(\frac{dx}{dt}) = \frac{g'(t)}{f'(t)}$. The arc length from $t = a$ to $t = b$ is $\int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$.

Polar Coordinates: $x = r\cos\theta$, $y = r\sin\theta$. $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$. The area bounded by $r = f(\theta)$, $\theta = a$, $\theta = b$ is $\int_a^b \frac{1}{2} r^2\, d\theta = \int_a^b \frac{1}{2}(f(\theta))^2\, d\theta$.