

Workshop 2, Math 152

1. Sketch the first quadrant region bounded by the parabola $y = x^2$, the tangent to the parabola at $(1, 1)$, and the x -axis, and calculate the area of this region.

2. Sketch the graph of $y_1 = \sin x$ and $y_2 = \cos x$ in the viewing window $[0, 2\pi] \times [-1, 1]$.

In this domain, find the area of the largest region that is bounded above by $y_1 = \sin x$ and below by $y_2 = \cos x$.

3. If f is a continuous function, verify, using substitution, that $2 \int_0^1 f(2x+1) dx = \int_1^3 f(t) dt$.

4. Suppose that $0 < a < b$. Let R be the region bounded by $y = 1/x$, $y = 0$, $x = a$, $x = b$.

(a) Find a vertical line $x = c$ that divides R into two subregions of equal area. (Your answer should express c as a function of a and b .)

(b) Use a picture to explain the inequality: $\sqrt{ab} \leq \frac{1}{2}(a+b)$.

5. Sketch the first quadrant regions:

(a) Bounded by $x^2 + y^2 = 1$ and $x + y = 1$

(b) Bounded by $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$.

Which of the two regions has the larger area? Explain your answer .

6. Let $f(x) = x^3$.

(a) Find the equation of the tangent line to the graph $y = f(x) = x^3$ at the point $(1, 1)$.

(b) Find the finite area bounded by $y = x^3$ and the tangent line to $y = x^3$ at the point $(1, 1)$.

7. Calculate the integrals:

$$(a) \int \frac{\tan(\sqrt{x})}{\sqrt{x}} dx \quad (b) \int \frac{\cos(\ln x)}{x} dx \quad (c) \int (2^x)^3 (3^x)^2 dx$$

8. A positive function f has the property that the area bounded by $y = f(x)$, $y = 0$, $x = 0$, $x = u$ is given by the formula $\frac{u^2}{u^2 + 1}$ for all $u > 0$. What is the function f ?

9. Sketch the region that is bounded above by the parabola $y = 1 - x^2$ and below by the x -axis.

(a) What is the area of this region?

(b) Find a horizontal line that divides the region into two subregions of equal area.