

Workshop 3, Math 152

1. A solid in three-dimensional space has its base in the first quadrant of the xy -plane. The base is bounded by the curves:

$$y = x^{1/3} \quad \text{and} \quad y = x^2.$$

Find the volume of the solid if

- (a) A plane perpendicular to the x -axis slices through the solid in a square, or
 - (b) A plane perpendicular to the y -axis slices through the solid in an equilateral triangle.
2. A triangle has vertices at the points $(2, 2)$, $(4, 2)$, and $(1, 3)$. Find the volume of the solid obtained by rotating the area within the triangle around the x -axis.
3. Suppose that the continuous function f is positive for all $x > 0$. Let R_a be the region bounded by $y = f(x)$, $y = 0$, $x = 0$, $x = a$, with a some positive real number.

Assume that the volume of the solid formed by rotating R_a around the x -axis is the same as the volume of the solid formed by rotating R_a around the y -axis, for all $a > 0$.

What is the function f ?

4. A region R is bounded by $y = 0$, $y = f(x) > 0$, $x = 0$, $x = t > 0$. If R is rotated around the x -axis, the volume of the resulting solid is $(\frac{\pi}{3})(t^3 + 3t)$.

If the same region is rotated around the y -axis, what is the volume of the solid obtained?

5. (a) Use your calculator to sketch the two functions

$$f(x) = x^2 - \frac{x^3}{3} \quad \text{and} \quad g(x) = \sin\left(\frac{\pi x}{3}\right)$$

on the interval $[0, 3]$. Note all the points of intersection as precisely as you can.

- (b) Compute $\int_0^3 (f(x) - g(x)) dx$ exactly. Just what does the value of this integral tell you about the areas of the regions between the two graphs? Explain your answer.

6. Let R be the first quadrant region bounded by the parabola $y = 2x - x^2$ and the x -axis. Find the constant m so that the line $y = mx$ divides R into two regions of the same area.