

Workshop 4, Math 152

1. Calculate four of the five indefinite integrals:

$$(a) \int x \sin(x^2) dx \quad (b) \int x^2 \sin(x) dx \quad (c) \int x^2 \sin(x^2) dx$$

$$(d) \int x^2 \sin(x^3) dx \quad (e) \int x^3 \sin(x^2) dx$$

2. Find all positive real numbers b so that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is exactly equal to 3. Use the numerical integration facility of your calculator to verify that your answer is accurate.

3. If p is any polynomial, verify that

$$\int p(x)e^x dx = (p(x) - p'(x) + p''(x) - p'''(x) + \dots)e^x + C$$

(Suggestion: Differentiate both sides.)

(a) What property of $p^{(n)}$, the n -th derivative of p , for n sufficiently large, makes this calculation possible?

(b) Calculate the following integrals:

$$(i) \int x^6 e^x dx \quad (ii) \int x^6 e^{2x} dx \quad (iii) \int e^{x^{1/7}} dx$$

4. Find the average value of $f(x) = |x|$ on the interval $[-1, 3]$.

5. Use the information about the function T and its first two derivatives, given in the table to the right, to calculate each of the following definite integrals.

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4

$$(a) \int_1^3 T'(x) dx \quad (b) \int_1^3 T''(x) dx \quad (c) \int_2^3 T(x)T'(x) dx \quad (d) \int_1^3 xT''(x) dx$$

6. Calculate the integral $\int_1^2 x \sin^{-1}(1/x) dx$.

7. Determine how large n has to be in order to approximate the integral $\int_0^1 e^{x^2} dx$, using the Midpoint Rule, with error at most $1/10^6$. Then, use this value of n to calculate the integral to this accuracy.