30 May 2008

Quiz 1: Section 6.3, Problem 36

Find the volume of the solid obtained by rotating the region enclosed by the graphs about the given axis.

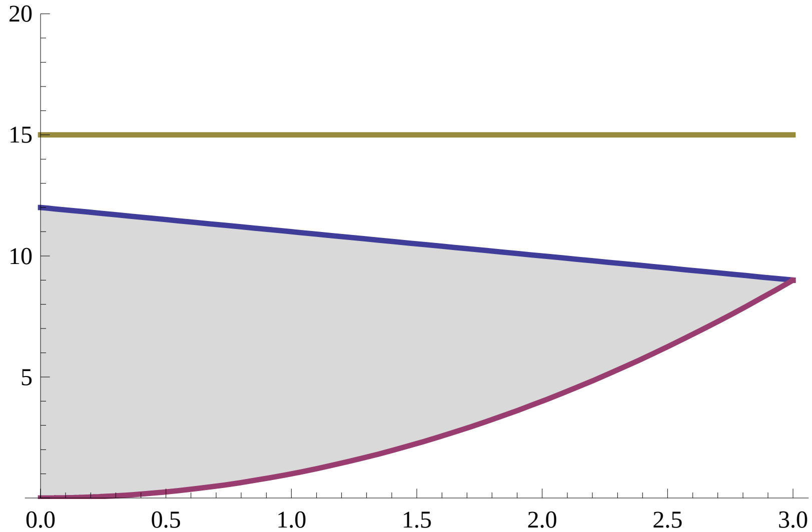
Region enclosed by:

$$y = x^2$$
$$y = 12 - x$$
$$x = 0$$

Rotated about:

$$y = 15$$

The region that is to be rotated and the rotating line is shown below:



The intersection point on the right-hand side is found by solving the two equations:

$$x^2 = 12 - x$$

yielding two solutions, $x = -4$ and $x = 3$. Since $x > 0$, then $x = 3$.
At some arbitrary x-value between 0 and 3, our cross-section is a washer with inner radius equal to $15 - (12 - x) = x + 3$ and an outer radius equal to $15 - x^2$. Therefore, the total area of our cross-section is $\pi(15 - x^2)^2 - \pi(x + 3)^2$ and our volume is

$$\begin{aligned} V &= \int_0^3 \pi(15 - x^2)^2 - \pi(x + 3)^2 \, dx \\ &= \pi \int_0^3 (225 - 30x^2 + x^4) - (x^2 + 6x + 9) \, dx \\ &= \pi \int_0^3 x^4 - 31x^2 - 6x + 216 \, dx \\ &= \frac{1953\pi}{5} \end{aligned}$$