

Quiz 16 - Math 152

Recall that we had proven in the workshop that

$$\sum_{k=1}^{n-1} f(k) \leq \int_1^n f(x) dx \leq \sum_{k=2}^n f(k)$$

Plug in $f(x) = \ln x$ and simplify to get

$$(n-1)! \leq n^n e^{-n} e \leq n!$$

Solution.

If we plug in $f(x) = \ln x$ we get

$$\begin{array}{lll} \sum_{k=1}^{n-1} \ln k \leq & \int_1^n \ln x dx \leq & \sum_{k=2}^n \ln k \\ \ln(n-1)! \leq & (x \ln x - x)|_1^n \leq & \ln(n!) \\ \ln(n-1)! \leq & n \ln n - n + 1 \leq & \ln(n!) \\ (n-1)! \leq & e^{n \ln n} e^{-n} e \leq & \ln(n!) \\ (n-1)! \leq & n^n e^{-n} e \leq & \ln(n!) \end{array}$$