

Quiz 2

Let A be the area bounded by the curves

$$\begin{aligned}y &= e^{x^2} \\y &= 0 \\x &= 0 \\x &= 5\end{aligned}$$

Find the volume when A is rotated about the y -axis.

Solution. We shall use cylindrical shells. At some x -value, the shell is a cylinder with radius x and height e^{x^2} . So the area of this cylinder is $2\pi x e^{x^2}$. Therefore, the volume we seek is

$$\int_0^5 2\pi x e^{x^2} dx$$

Using the substitution $u = x^2$ we obtain $du = 2x dx$ so

$$\begin{aligned}\int_0^5 2\pi x e^{x^2} dx &= \int_0^5 \pi e^u du \\&= \pi(e^u)|_0^5 \\&= \pi(e^{x^2})|_0^5 \\&= \pi(e^{25} - e^0) \\&= \pi(e^{25} - 1)\end{aligned}$$

and we're done!