

Quiz 4 - Math 152

1. (3pt) Find $\int x^4 \ln x \, dx$.
2. (7pt) Find $\int \cos(\ln x) \, dx$.

Solutions.

1. We use integration by parts, with

$$\begin{aligned} u &= \ln x & v &= \frac{x^5}{5} \\ du &= \frac{1}{x} \, dx & dv &= x^4 \, dx \end{aligned}$$

and we get

$$\int x^4 \ln x \, dx = \frac{x^5 \ln x}{5} - \frac{1}{5} \int x^4 \, dx = \frac{x^5 \ln x}{5} - \frac{x^5}{25}$$

2. Again, we use integration by parts, with

$$\begin{aligned} u &= \cos(\ln x) & v &= x \\ du &= -\frac{\sin(\ln x)}{x} \, dx & dv &= 1 \, dx \end{aligned}$$

and we get

$$\int \cos(\ln x) \, dx = x \cos(\ln x) + \int \sin(\ln x) \, dx$$

we use integration by parts again, this time with

$$\begin{aligned} u &= \sin(\ln x) & v &= x \\ du &= \frac{\cos(\ln x)}{x} \, dx & dv &= 1 \, dx \end{aligned}$$

and we get

$$\int \sin(\ln x) \, dx = x \sin(\ln x) - \int \cos(\ln x) \, dx$$

So we can combine it all and solve for $\int \cos(\ln x) \, dx$ to get

$$\int \cos(\ln x) \, dx = \frac{x}{2}(\cos(\ln x) + \sin(\ln x))$$