642:587 Convex and Discrete Geometry, Homework 4

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Homework 4 must be handed in class on Wednesday, 11/02/05

You are required to hand in written solutions for 6 problems.

Problems from the book

Section 6.1: 2; Section 6.2: $2ab, 5$; Section 6.3: $2, 4, 5$.

Section 6.4: $2, 3$; Section 7.1: $1, 3, 4, 5, 7$.

Section 7.3: $1, 2$; Section 7.4: 1; Section 7.5: $1, 2$.

Problem 1: Let P be a simple polygon in the plane with $n \geq 3$ vertices. Prove that every triangulation of P has at least two "ears", namely triangles that share two edges with P. (Hint: One way to show this is via (circular) Davenport-Schinzel sequences of order 2. Given a triangulation of P, walk around the boundary of P in ccw order, and construct a sequence as follows: when reaching a new vertex v of P, add to the sequence all triangles incident to v in cw order about v, except for the triangle incident to the edge of P that led you to v.)

Problem 2: Geometric permutations Let $\mathcal{F}$ be a collection of n pairwise disjoint compact convex sets in the plane. Assume that the sets in $\mathcal{F}$ are labeled from 1 to n. We say that a directed line transversal $\overrightarrow{l}$ induces a permutation $(i_1, i_2, \ldots, i_n)$ if $\overrightarrow{l}$ intersects the sets of $\mathcal{F}$ in this order. An undirected line transversal l induces a *geometric permutation* consisting of $(i_1, i_2, \ldots, i_n)$ and its reverse, if one of the two directed lines coinciding with l intersect the sets of $\mathcal{F}$ in the order $(i_1, i_2, \ldots, i_n)$. Let $n \geq 4$. Show that the maximum number of geometric permutations is $2n - 2$. Can you show that this bound is tight? (Hint: Try to position two large discs and $n - 2$ horizontal segments in an appropriate way.) This is a result of Edelsbrunner and Sharir (1990) which is just a sample from numerous beautiful results on the number of geometric permutations for various families of geometric objects.