642:587 Convex and Discrete Geometry, Homework 5

Radoš Radoičić

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Homework 5 must be handed in class on Monday, 11/14/05

You are required to hand in written solutions for 6 problems.

Problems from the book

Section 1.3: $2, 3, 4, 5, 8$; Section 8.1: $1, 2$.

Section 8.3: $1, 2, 4, 6$. Section 9.2: 1.

Section 9.3: $1, 3c, 4$.

Problem 1: Given n parallel line segments in the plane. Prove that if for each three of them there exists a common line transversal, then all of them have a common line transversal.

Problem 2: Show that if every two rectangles of a family of axis-parallel rectangles have a common point, then all the rectangles of the family have a common point.

Problem 3: Show that if a family of arcs on the circle is such that every two of the arcs have a common point, then there is an antipodal pair of points such that each arc of the family includes at least one point of the pair.

Problem 4: Let $\mathcal{F}$ be a family of n circular discs in the plane. Prove that there exist two subfamilies $\mathcal{F}_1$ and $\mathcal{F}_2$ of $\mathcal{F}$, each of size cn ($c > 0$ is some constant) such that either every disc in $\mathcal{F}_1$ intersects all discs in $\mathcal{F}_2$, or no disc in $\mathcal{F}_1$ intersects any disc in $\mathcal{F}_2$.

Problem 5: Show that there exists a positive constant $\delta > 0$ such that every family $\mathcal{L}$ of n straight skew lines in 3-space (no line is vertical) contains a tournament on at least n^δ lines, that is a sequence $l_1, l_2, \ldots, l_k$ of $k \geq n^\delta$ lines, such that l_i passes above l_j for all $i < j$.