On the decay of crossing numbers

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Abstract

The crossing number $\mathrm{cr}(G)$ of a graph G is the minimum number of crossings over all drawings of G in the plane. In 1993, Richter and Thomassen conjectured that there is a constant c such that for every graph G with crossing number k, there is an edge e of G such that $\mathrm{cr}(G-e) \geq k - c\sqrt{k}$. They showed only that G always has an edge e with $\mathrm{cr}(G-e) \geq \frac{2}{5}\mathrm{cr}(G) - O(1)$. We give a proof that for every connected graph G with $n \geq 2$ vertices edges, and every edge e of G, we have $\mathrm{cr}(G-e) \geq \mathrm{cr}(G) - 2m + n/2 + 1$. This confirms the Richter-Thomassen conjecture for dense graphs. We also prove that for every fixed $\epsilon > 0$, there is a constant n_0 depending on ϵ such that if G is a graph with $n > n_0$ vertices and $m > n^{1+\epsilon}$ edges, then G has a subgraph G' with at most $(1 - \frac{1}{24\epsilon} + o(1))m$ edges such that $\mathrm{cr}(G') \geq \frac{1}{28}\mathrm{cr}(G)$. Joint work with Csaba Tóth.

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