Math 477, Homework 5, due 3/23/06

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You are required to hand in any 13 of the following problems.

At least two have to be theoretical exercises.

The solutions should be clearly written explanations, not just numbers.

Problems from the book:

Chapter 5 – Problems: $2, 5, 8, 13, 15, 16, 17, 19, 21$.

Chapter 5 – Problems: $25, 27, 28, 32, 34, 36, 38, 40$.

Chapter 5 – Theoretical Exercises: $4, 8, 12, 15, 16, 18, 20, 29, 30$.

Problem 1: (three problems worth) This is one version of the famous "Monty Hall" game show question. There are three doors. You know there is a prize behind one of them, and nothing behind the other two. The game show host tells you that you shall receive whatever is behind the door of your choice. However, before you choose, he promises that rather than immediately opening the door of your choice to reveal its contents, he will open on of the other two doors to reveal that it is empty. He will then give you the option to switch your choice. You may assume that the host is totally impartial - not malicious in any way. You choose Door 3. He opens Door 2 and reveals that it is empty. You now know that the prize lies behind either Door 3 or Door 1. Should you switch your choice to Door 1?

Problem 2: (two problems worth) When five points are chosen uniformly at random from the interval $[1, 2]$, what is the distribution of the natural logarithm of the smallest point?

Problem 3: (two problems worth) A gangster stands 10m from an infinitely long straight wall. The gangster fires a gun horizontally in a completely random direction toward the wall. Compute the distribution of the point on the wall where the bullet hits. Do the same for the distance from the bullet to the point of the wall closest to the gangster.