Math 250, Homework 2, Due 02/04/05

Radoš Radoičić

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You do not have to do the bonus problems.

Problem 0 Do the following problems from the book:

$$\text{Section } 1.6 : 20, 28, 37, 39, 42, 45, 47.$$

Problem 1 Find the vector x from $3(a_1 - x) + 2(a_2 + x) = 5(a_3 + x)$, where $a_1 = (2, 5, 1, 3)$, $a_2 = (10, 1, 5, 10)$, and $a_3 = (4, 1, -1, 1)$.

Problem 2 Solve the following system of equations:

$$\begin{array}{ccccccccccc} 6x_1 & + & 3x_2 & + & 2x_3 & + & 3x_4 & + & 4x_5 & = & 5 \\ 4x_1 & + & 2x_2 & + & x_3 & + & 2x_4 & + & 3x_5 & = & 4 \\ 4x_1 & + & 2x_2 & + & 3x_3 & + & 2x_4 & + & x_5 & = & 0 \\ 2x_1 & + & x_2 & + & 7x_3 & + & 3x_4 & + & 2x_5 & = & 1 \end{array}$$

Problem 3 Find the rank of the following matrix:

$$\begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & -6 \end{pmatrix}$$

Problem 4 Find values λ such that the matrix has the smallest possible rank:

$$\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix}$$

Problem 5 Determine if the following vectors are linearly independent: $a_1 = (1, 0, 0, 2, 5)$, $a_2 = (0, 1, 0, 3, 4)$, $a_3 = (0, 0, 1, 4, 7)$, and $a_4 = (2, -3, 4, 11, 12)$.

Problem 6 Find all the values of λ for which $b = (1, 3, 5)$ is in the span of the following vectors: $a_1 = (3, 2, 5)$, $a_2 = (2, 4, 7)$, $a_3 = (5, 6, \lambda)$.

Problem 7 Find all the maximal linearly independent subsets of the following set of vectors: $a_1 = (4, -1, 3, -2)$, $a_2 = (8, -2, 6, -4)$, $a_3 = (3, -1, 4, -2)$, and $a_4 = (6, -2, 8, -4)$.

Bonus problem 1: worth 5 other problems Discuss the existence and the number of solutions of the following system depending on the values of parameters a, b, c, and d.

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ ax & + & by & + & cz & = & d \\ a^2x & + & b^2y & + & c^2z & = & d^2 \end{array}$$

Bonus problem 2: worth 3 other problems Prove that $\text{rank}(A+B) \leq \text{rank}(A)+\text{rank}(B)$, for every two $m \times n$ matrices A and B.