Math 250, Homework 3, Due 02/11/05

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You do not have to do the bonus problems.

Problem 0 Do the following problems from the book:

$$\text{Section } 1.7: 17, 33, 40, 43, 45, 50.$$

$$\text{Section } 2.1: 22, 35, 40, 51, 55, 57.$$

Problem 1 An airline serves five cities, say A, B, C, D, and H, where H is the hub city. One can take a direct flight from H to any other city, as well as from every city to H. One can also take direct flights from A to C, from C to D, from D to B, and from B to A. How many routes from city A to city B are there that require no more than four flights. Use linear algebra!

Problem 2 If $A = [a_{ij}(t)]$ is a matrix whose entries are functions of a variable t, the derivative of A with respect to t is defined to be the matrix of derivatives, i.e.

$$\frac{dA}{dt} = [\frac{da_{ij}}{dt}].$$

Derive the product rule for differentiation:

$$\frac{d(AB)}{dt} = \frac{dA}{dt}B + A\frac{dB}{dt}.$$

Problem 3 For each square $n \times n$ matrix A, explain why it is impossible to find a square $n \times n$ matrix X such that $AX - XA = I$. Hint: Use 2.1.55.

Bonus Problem: Can replace 3 other problems A particular electronic device consists of a collection of switching circuits that can be either in an ON state or an OFF state. These electronic switches are allowed to change state at regular time intervals called clock cycles. Suppose that at the end of each clock cycle, 30% of the switches currently in the OFF state change to ON, while 90% of those in the ON state revert to the OFF state.

1. Show that the device approaches an equilibrium in the sense that the proportion of switches in each state eventually becomes constant, and determine these equilibrium proportions.
2. Independent of the initial proportions, about how many clock cycles does it take for the device to become essentially stable?

Bonus problem: Can replace 4 other problems For the matrix A:

$$\begin{pmatrix} 1 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{pmatrix}$$

determine A^{300}. Hint: Blocks!!!