Math 250, Homework 4, Due 02/18/05

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You do not have to do the bonus problems.

Problem 0 Do the following problems from the book:

Section 2.3 : 8, 16, 18, 19, 28, 31.

Section 2.4 : 10, 15, 21, 27, 33, 34, 51, 52.

Problem 1 Calculate A^n, B^n and C^3, where A, B, and C are these matrices respectively.

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Problem 2 Consider the block matrices

$$\begin{pmatrix} A_{r\times r} & C_{r\times s} \\ R_{s\times r} & B_{s\times s} \end{pmatrix}$$

When the indicated inverse exists, the matrix defined by $S = B - RA^{-1}C$ is called a *Schur complement* of A. If A and S are both invertible, verify that

$$\begin{pmatrix} A & C \\ R & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + A^{-1}CS^{-1}RA^{-1} & -A^{-1}CS^{-1} \\ -S^{-1}RA^{-1} & S^{-1} \end{pmatrix}$$

Bonus Problem: Can replace 4 other problems Find the inverses of the following $n \times n$ matrices:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & \dots & n-1 & n-2 \\ 0 & 1 & 2 & 3 & \dots & n-2 & n-1 \\ 0 & 0 & 1 & 2 & \dots & n-3 & n-2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}$$