

# Math 250, Homework 5, Due 03/04/05

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**You can choose between Problem 2 and 3, or do both for extra credit.**

**Problem 0** Do the following problems from the book:

Section 3.1 : 18, 25, 47, 48.

Section 3.2 : 15, 17, 38, 43, 45.

**Problem 1:** Calculate the determinant of the following matrix. Assume that for all  $i$ , we have  $x \neq a_i$ .

$$\begin{pmatrix} a_1 & x & x & \dots & x \\ x & a_2 & x & \dots & x \\ x & x & a_3 & \dots & x \\ \dots & \dots & \dots & \dots & \dots \\ x & x & x & \dots & a_n \end{pmatrix}$$

**Problem 2: Cofactor expansions and elementary operations, sometimes with columns!**  
Calculate the determinants of the following matrices:

$$\begin{pmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & x & \dots & x & x \\ 1 & x & 0 & \dots & x & x \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x & x & \dots & 0 & x \\ 1 & x & x & \dots & x & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 + a_1 b_1 & 1 + a_1 b_2 & \dots & 1 + a_1 b_n \\ 1 + a_2 b_1 & 1 + a_2 b_2 & \dots & 1 + a_2 b_n \\ \dots & \dots & \dots & \dots \\ 1 + a_n b_1 & 1 + a_n b_2 & \dots & 1 + a_n b_n \end{pmatrix}$$

**Problem 3: Recursive formulas** Calculate the determinants of the following matrices.  
 (Hint: With the second one, you should get Fibonacci numbers. You do not have to solve the recursive formula and get the closed form.)

$$\begin{pmatrix} 3 & 2 & 0 & 0 & \dots & 0 & 0 \\ 1 & 3 & 2 & 0 & \dots & 0 & 0 \\ 0 & 1 & 3 & 2 & \dots & 0 & 0 \\ 0 & 0 & 1 & 3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 3 & 2 \\ 0 & 0 & 0 & 0 & \dots & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}$$