

# Lower bounds for algorithms in abstract cubes

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## Abstract

An abstract objective function (AOF) is a real valued function on the vertices of a polyhedron such that every face has exactly one local maximum. Naturally, a generic enough linear function over a polyhedron satisfies this property. AOF is a very useful model for Linear Programming as the current best algorithms whose running time depends only on the dimension and the number of facets of the polytope do run in this model. The oldest and still widely used approach to maximize linear functions over a polytope is the simplex method, where we follow a path on the edges of the polytope, going from vertex to vertex and always improving the objective function value and eventually hitting the maximum. Such algorithms have great practical success, still theoretical worst case bounds exist for almost all deterministic variants. The behaviour of the simplest randomized simplex algorithm (the one selecting uniformly at random among improving neighbors) is not known. In the talk we describe a technique which can successfully be applied to obtain lower bounds for the performance of several algorithms maximizing abstract objective functions on the hypercube. In particular, we discuss the random simplex algorithm and a couple of other deterministic processes which not necessarily follow a path. Based on joint works with Jiří Matoušek and Ingo Schurr.

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