

### Exam Formula sheet for Math 152

$$\begin{aligned}\sin(0) &= 0; & \sin\left(\frac{\pi}{6}\right) &= \frac{1}{2}; & \sin\left(\frac{\pi}{4}\right) &= \frac{\sqrt{2}}{2}; & \sin\left(\frac{\pi}{3}\right) &= \frac{\sqrt{3}}{2}; & \sin\left(\frac{\pi}{2}\right) &= 1; & \sin(\pi) &= 0 \\ \cos(0) &= 1; & \cos\left(\frac{\pi}{6}\right) &= \frac{\sqrt{3}}{2}; & \cos\left(\frac{\pi}{4}\right) &= \frac{\sqrt{2}}{2}; & \cos\left(\frac{\pi}{3}\right) &= \frac{1}{2}; & \cos\left(\frac{\pi}{2}\right) &= 0; & \cos(\pi) &= -1\end{aligned}$$

$$\cos^2 x + \sin^2 x = 1, \quad 1 + \tan^2 x = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x, \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x)), \quad \sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C, \quad \int \csc x \, dx = \ln |\csc x - \cot x| + C$$

If  $T_N, M_N, S_N$  are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)];$$

$$M_N = \Delta x[f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_i = (x_{i-1} + x_i)/2;$$

$$S_N = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)].$$

If  $I = \int_a^b f(x)dx$  then:

$$|T_N - I| \leq \frac{K_2(b-a)^3}{12N^2}, \quad |M_N - I| \leq \frac{K_2(b-a)^3}{24N^2}, \quad |S_N - I| \leq \frac{K_4(b-a)^5}{180N^4}.$$

$$\text{length} = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx, \quad \text{surface area} = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx$$

$$\text{For parametric curves: length} = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$$

$$\text{For polar curves: length} = \int_a^b \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta; \quad \text{area} = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

The  $n$ th Taylor Polynomial of  $f(x)$  with center  $c$  is  $T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(c)}{k!} (x - c)^k$

$$\text{If } |f^{(n+1)}(u)| \leq K \text{ for all } u \text{ between } c \text{ and } x, \text{ then } |f(x) - T_n(x)| \leq K \frac{|x - c|^{n+1}}{(n+1)!}$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \cdots \text{ if } |x| < 1.$$