Sec 01-06 **Calculus 251 Formula Sheet** **FALL 2008**

Trig: $\sin^2 x + \cos^2 x = 1$ $\sec^2 x = 1 + \tan^2 x$ $\sin^2 x = \dfrac{1 - \cos 2x}{2}$ $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

Angle x: $0, 2\pi$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$
$\sin x$: 0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\cos x$: 1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	0
$\tan x$: 0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	*undef*	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0	*undef*

Line: $x = x_o + at$ $y = y_o + bt$ $z = z_o + ct$ where the direction vector is $\langle a, b, c\rangle$

Plane: $a(x - x_o) + b(y - y_o) + c(z - z_o) = 0$ where $\mathbf{n} = \langle a, b, c\rangle$

Quadric Surfaces: * note: if $a = b = c$, then the ellipsoid is a sphere.

Ellipsoid: * $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ Hyperboloid of One Sheet: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$

Hyperboloid of Two Sheets: $-\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ Cone: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z^2}{c^2}$

Elliptic Paraboloid: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z}{c}$ Hyperbolic Paraboloid: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \dfrac{z}{c}$

Coordinate Conversion: From Cylindrical to Rectangular: $x = r\cos\theta$ $y = r\sin\theta$ $z = z$

From Rectangular to Cylindrical: $r^2 = x^2 + y^2$ $\tan\theta = \dfrac{y}{x}$ $z = z$

From Spherical to Rectangular: $x = \rho\sin\phi\cos\theta$ $y = \rho\sin\phi\sin\theta$ $z = \rho\cos\phi$

From Rectangular to Spherical: $\rho^2 = x^2 + y^2 + z^2$ $\tan\theta = \dfrac{y}{x}$ $\phi = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

From Spherical to Cylindrical: $r = \rho\sin\phi$ $\theta = \theta$ $z = \rho\cos\phi$

From Cylindrical to Spherical: $\rho^2 = r^2 + z^2$ $\theta = \theta$ $\phi = \cos^{-1}\left(\dfrac{z}{\sqrt{r^2 + z^2}}\right)$

Jacobian for Polar and Cylindrical integration: r, for Spherical integration: $\rho^2\sin\phi$

Useful Formulas: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$ $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$ $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

$\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|}$ $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|^2}\mathbf{a}$ $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$ $\kappa = \dfrac{|f''(x)|}{\left(1 + [f'(x)]^2\right)^{\frac{3}{2}}}$

$D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$ where $\mathbf{u}$ is a unit vector, $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ where $a_T = \mathbf{a}\cdot\mathbf{T}$ $a_N = \|\mathbf{a} \times \mathbf{T}\|$

Gradients: $\nabla f(x, y) = f_x\mathbf{i} + f_y\mathbf{j}$ $\nabla f(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$

Linearization: $L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$

$L(x, y, z) = f(a, b, c) + f_x(a, b, c)(x - a) + f_y(a, b, c)(y - b) + f_z(a, b, c)(z - c)$

Total Differential: $dz = f_x(a, b)dx + f_y(a, b)dy = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

$$dw = f_x(a, b, c)dx + f_y(a, b, c)dy + f_z(a, b, c)dz = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

Implicit Function Thm: For $F(x, y, z) = 0$ $\dfrac{\partial z}{\partial x} = -\dfrac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$ $\dfrac{\partial z}{\partial y} = -\dfrac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$

Second Partials Test: $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.

b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.

c. If $D < 0$ then (a,b) gives a saddle point of f. If $D = 0$ this test is inconclusive.

Arc Length for a curve C given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k} \quad a \le t \le b:$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b \|\mathbf{r}'(t)\| dt$$

Line Integral for a scalar function f;

$$\int_C f(x,y,z)\, ds = \int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt = \int_a^b f(\mathbf{r}(t))\, \| \mathbf{r}'(t) \|\, dt$$

Line Integral for Vector Field F: $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = \int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\ dt$

Fundamental Theorem for Line Integrals $\displaystyle\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$

Component Test for Conservative Force Fields: $\dfrac{\partial R}{\partial y} = \dfrac{\partial Q}{\partial z}$, $\dfrac{\partial P}{\partial z} = \dfrac{\partial R}{\partial x}$, AND $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$

curl F $= \nabla \times \mathbf{F} = \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x}\right)\mathbf{j} + \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right)\mathbf{k}$

div F $= \nabla \cdot \mathbf{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z}$

Green's Theorem : $\displaystyle\oint_C P\, dx + Q\, dy = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

Area of a surface given by $z = g(x,y)$, $(x,y) \in D:$ $\displaystyle A(S) = \int\int_D \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1}\ dA$

Area of smooth surface S parameterized by $\mathbf{r}$(u,v)= $x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k} \quad (u,v) \in D$ is given by $\displaystyle A(S) = \int\int_D \| \mathbf{r}_u \times \mathbf{r}_v \|\, dA$

Surface Integral for a surface given by $z = g(x,y)$**:**

$$\int\int_S f(x,y,z)\, dS = \int\int_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\ dA$$

Surface Integral for a surface parameterized by $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$
$(u,v) \in D$

$$\int\int_S f(x,y,z)\, dS = \int\int_D f(\mathbf{r}(u,v))\, \| \mathbf{r}_u \times \mathbf{r}_v \|\, dA$$

Stokes' Theorem : $\displaystyle\oint_C \mathbf{F} \cdot d\,\mathbf{r} = \int\int_S \textbf{curl } \mathbf{F} \cdot d\mathbf{S} = \int\int_S \textbf{curl } \mathbf{F} \cdot \mathbf{n}\ dS$

Divergence Theorem : $\displaystyle\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int_S \mathbf{F} \cdot \mathbf{n}\ dS = \int\int\int_E \textbf{div } \mathbf{F}\ dV$