

## Hints for Workshop 10

1. Determine whether the series are absolutely convergent, conditionally convergent, or divergent. Explain.

$$\begin{aligned}
 \text{a)} \sum_{n=3}^{\infty} \frac{2\sqrt{n}(-1)^n}{5n^2 - 10n + 20} , \quad & \text{b)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3n-1)}{n^3 - 1} , \quad & \text{c)} \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n - 1} \\
 \text{d)} \sum_{n=0}^{\infty} \frac{1 - e^{n-1}}{2 + e^n} , \quad & \text{e)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1 - 3n} , \quad & \text{f)} \sum_{n=1}^{\infty} \frac{2^n - 1}{ne^n} \\
 \text{g)} \sum_{n=1}^{\infty} \frac{\cos n}{n\sqrt{n} + 1} , \quad & \text{h)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{2n^2 - 1}} , \quad & \text{k)} \sum_{n=1}^{\infty} \frac{(-1)^n n^{4/3}}{n!} \\
 \text{l)} \sum_{n=1}^{\infty} \frac{(2n)!}{2^n (n!)^2} \quad & \text{m)} \sum_{n=1}^{\infty} \frac{3^n}{n!} \quad & \text{n)} \sum_{n=1}^{\infty} ne^{-n} \quad & \text{p)} \sum_{n=1}^{\infty} 2^n e^{-n^2}
 \end{aligned}$$

**Hints:** The series a)...k) are alternating: have the form  $\sum(-1)^n b_n$  or  $\sum(-1)^{n-1} b_n$  with  $b_n > 0$ .

Do series have  $b_n \rightarrow 0$ ? Example **d)** does not since  $b_n \sim \frac{-e^{n-1}}{e^n} = -e^{-1}$  so diverges.

All other series do have  $b_n \rightarrow 0$ . Hence, next question is: is the sequence  $b_n$  decreasing? This is clearly the case for series **e)** and **h)**: so these two converge.

For the rest:

**a)**  $b_n \sim 2\sqrt{n}/5/n^2 = 2/5 1/n^{3/2}$ . Since the harmonic series  $\sum 1/n^{3/2}$  converges, then we expect our series to converge absolutely. Use limit comparison test (compare  $\sum b_n$  to  $\sum 1/n^{3/2}$ ).

**b)** We have  $b_n \sim 3/n^2$  so as above ( $\sum 1/n^2$  converges, then we expect our series to converge absolutely. Use limit comparison test: compare  $\sum b_n$  to  $\sum 1/n^2$ .)

**c)** As before: compare  $\sum b_n$  to  $\sum (2/3)^n$ .

**f)** As before: compare  $\sum b_n$  to  $\sum_{n=1}^{\infty} (2/e)^n/n$ . The latter converges since is less than  $\sum_{n=1}^{\infty} (2/e)^n$ .

**g)** Is not alternating, but not a series with positive terms either (because of  $\cos n$ ). Only hope is to test for absolute convergence. This is simple since  $|\cos| \leq 1$  hence  $|a_n| < 1/n^{3/2}$  converges.

**k)** Alternating. Since  $n^{4/2} \ll n!$  we expect absolute convergence. Factorial asks for ratio test.

**l)** Factorial asks for ratio test.

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- m)** Series converges and sum equals  $e^3$ .
- n)** Order of growth: convergence is decided by the  $e^{-n}$ . Try either root or ratio test.
- p)** Try either root or ratio test.