Rodica D. Costin

Research Statement

My research is focused on differential equations, with emphasis on the study of singularities and asymptotic behavior of solutions, regularity, normal forms, the study of non-integrable systems and connection to chaos.

I am applying new methods—analysis of equations in Borel space (after inverse Laplace transform) and a theory of generalized Borel summation of general formal solutions (transseries).

These methods made possible to uncover, and fully describe, *general structures of singularities* that solutions develop in the presence of irregular singularities of equations. In a paper in Inventiones [1] we show that the regular distribution of spontaneous singularities observed in many concrete examples is in fact generic, and introduce practical methods to predict the position and type of these singularities.

Generalizing Borel-Laplace methods to operator theory provided new methods in partial differential equations.

Time-dependent Schrödinger equation

There is a vast amount of recent literature on ionization of atoms in high amplitude fields, experimental as well as numerical and theoretical, based on approximate semiclassical analysis [2], but before [3] there was no detailed rigorous mathematical study of the time-behavior of systems with spatial structure under perturbations of arbitrary strength.

In a series of papers [4], [5], [6], [7], which study models from simple, to quite general, we rigorously solve classes of equations with time-periodic potentials. Solutions of the time-dependent Schrödinger equation are shown to have representations as Borel-summed series of powers and small exponential terms in t. The important question answered using this structure was finding conditions for ionization, or show its opposite, stabilization (conditions related to absence or presence of the discrete spectrum in the Floquet theory).

It turns out that the quantities calculated with the models compare very well qualitatively with recent experimental results and with approximate calculations and revealed universal structures.

Important extensions are now in work, in particular to 3-dimensional models with slower decaying, realistic potentials. In fact, core results in this program already appear in [6]. Of high priority and interest are Coulomb potentials, either parametrically perturbed or in interaction with electromagnetic field.

Extensions to other partial differential equations The study of Schrödinger's equation revealed that fundamental features are encoded in the exponentially small terms of expansions of solutions for large t. I have recently studied these terms in more general setting [8] (and their implications to the regularity of solutions [9]). New, general features were uncovered and I am interested in pursuing these ideas. In particular I am applying them to problems in hydrodynamics (a paper is now being prepared for publication); it is very exciting to see the results produced by the new methods.

Nonintegrability and chaos

I am also interested in understanding qualitative behavior of dynamical systems from the point of view of integrability, or, moreover, its absence, and its connection to chaos. These questions are fundamental in countless applications in physics, biology, engineering, and many other fields. Important, deep results have been obtained along the time. However, basic questions remain open, and there is no clear-cut, widely agreed and general understanding of when a system should be considered non-integrable, what practical testing methods can be used, and how this property is related to chaos.

I was introduced to this problem area as a graduate student by my advisor, Martin Kruskal. He noted that obstructions to integrability of a differential equation are encoded in the multi-valuedness of solutions, and proposed a practical methodology in testing for nonintegrability.

As part of my graduate research, and continuing throughout the years, I have been interested in developing a rigorous and general framework for Kruskal's approach, and exploring its consequences. I have found rigorous criteria for nonintegrability (in a very general sense) for generic (non-resonant, non-quasiresonant) systems with a fixed point and regular singularities [10], [11], [12], [13], [14].

I have been also actively involved in *research activities with undergraduate students*, part of the REU program at Rutgers, and this activity is being finalized with a paper which already appeared [15], and two papers in preparation [16],[17].

References

[1] O. Costin, R. D. Costin *On the location and type of singularities of nonlinear differential systems*, Invent. Math. 145, 425-485 (2001)

[2] P. M. Koch, Acta Physica Polonica A, **93** No. 1, 105–133 (1998).

[3] O. Costin, J. L. Lebowitz, A. Rokhlenko *Ionization of a Model Atom: Exact Results and Connection with Experiment*, J. Phys. A: Math. Gen. 33 (2000) 6311–6319.

[4] O. Costin, R. D. Costin, J. L. Lebowitz, A. Rokhlenko *Asymptotic evolution of a model quantum system under time periodic forcing: conditions for complete ionization*, Comm. Math. Phys. 221, 1, pp 1-26 (2001)

[5] O. Costin, R. D. Costin, J.L.Lebowitz, A. Rokhlenko *Nonperturbative analysis of a model quantum system under time periodic forcing*, Comptes rendus de l'Académie des sciences - Série I - Mathématique, 332 5, pp.405-411 (2001)

[6] O. Costin, R. D. Costin, J. L. Lebowitz *Time asymptotics of the Schrödinger wave function in time-periodic potentials*, J.Stat.Phys., special issue dedicated to Elliott Lieb on the occasion of his 70th birthday, **1-4** 283-310 (2004)

[7] O. Costin, R. D. Costin, J. L. Lebowitz *Time behavior and ionization conditions for time-periodic quantum models* (in preparation)

[8] R.D. Costin *Power and exponential-power series solutions of evolution equations*, to appear in Annales de la Faculte des Sciences de Toulouse (expected Nov.2004)

[9] O. Costin, R.D. Costin, *Failure of analytic hypo-ellipticity in a class of partial differential operators*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 2 (2003), no. 1, 21–45

[10] R. D. Costin *Applications of the Poly-Painlevé Test*, Thesis, Rutgers University, Oct. 1997

[11] R. D. Costin *Integrability Properties of a Generalized Lamé Equation; Applications to the Hénon-Heiles System*, Methods Appl.An. 4 no 2, 113-123 (1997)

[12] R. D. Costin *Integrability Properties of Nonlinearly Perturbed Euler Equations*, Nonlinearity, vol.10,no 4, 905-924 (1997)

[13] R. D. Costin, M.D. Kruskal *Nonintegrability Criteria for a Class of First Order Differential Equations with Several Singular Points*, Nonlinearity 16 (2003), no. 4, 1295–1317

[14] R. D. Costin, *Normal forms of differential equations in a region containing two regular singular points* (in preparation)

[15] O. Costin, R.D. Costin, M. Kohut, *Rigorous estimates of Stokes constants for some nonlinear differential equations with rank one irregular singularity*, to appear in Proc. Royal Soc. London

[16] R.D. Costin, M. Grabchak, M. Kohut (in preparation)

[17] O. Costin, R.D. Costin, J. McClain, J. Smallik (in preparation)