Math 428 - Solutions to (some) review problems
for Exam #1 - October 9, 2009

These are solutions to problems that require diagrams (and hence require a scanner to turn into pdf format). Solutions to other problems will be posted separately.

#2 (a) \((6,5,3,2,2,1,1,1)\) has an odd number of odd entries, so it is not graphical.

(b) \((7,6,5,4,4,4,4,3)\) is not graphical for the same reason.

(c) \((7,6,5,4,4,3,2,1)\) is graphical if and only if \((5,4,3,3,2,1,0)\) is graphical.

Thus, it is graphical if and only if \((3,2,2,1,0,0)\) is graphical. This is graphical, for example:

```
 o--o--o--o
```

Then

```
| o--o--o--o
```

has degree sequence \((5,4,3,3,2,1,0)\)
and

has degree sequence \((7, 6, 5, 4, 4, 3, 2, 1)\)

\((d)\) \((7, 6, 6, 5, 4, 3, 2, 1)\) is graphical if and only if \((5, 5, 4, 3, 2, 1, 0)\) is graphical.

This is graphical if and only if \((4, 3, 2, 1, 0, 0)\) is graphical. But \((4, 3, 2, 1, 0, 0)\) is not graphical. Since this is not graphical, neither is \((7, 6, 6, 5, 4, 3, 2, 1)\).

#4 Let \(G\) be the given graph. It has two vertices of degree 1 and 4 vertices of degree 2. Thus a 3-regular graph.
containing $G$ as an induced subgraph must contain at least 2 new vertices (for every vertex of order 1 must be adjacent to two new vertices) and at least 8 new edges (1 for each vertex of order 2 and 2 for each vertex of order 1). But if there are 8 new edges there must be at least $\frac{8}{3}$ new vertices. Thus there must be at least 3 new vertices, so a 3-regular graph containing $G$ as an induced subgraph must have at least 11 vertices. Since the number of vertices in a 3-regular graph must be even (since 3 is odd) there must be at least 12 vertices. Here is such a graph at order 12.
# 8

(a) 

(b) (which is just C₅ again)

(c)
(d)

Fig. 10 (a)

Kruskal

Prim

This is arbitrary
# 10 (b)

Kruskal

Prim

this is arbitrary