#1 In each of the following graphs find all cut-vertices and all bridges.

a) ![Graph A]

b) ![Graph B]

c) ![Graph C]

#2 Find all blocks of

![Graph D]


#3 Find a minimum vertex cut and a minimum edge cut of each of the following graphs.

\[ a \]

\[ b \]

These are copies of \( K_6 \)
#4 Does the graph $G_n$ (for $n=1, 2, 3$) contain an Eulerian circuit? Why or why not? If it contains an Eulerian circuit find one.

Does the graph $G_n$ (for $n=1, 2, 3$) contain an Eulerian trail? Why or why not? If it contains an Eulerian trail find one.

a) $G_1$

b) $G_2$

c) $G_3$
#5 Does the graph $G_n$ (for $n=1,2,3$) contain a Hamiltonian cycle? If not, why not? If so, find one.

(a) $G_1$

(b) $G_2$

(c) $G_3$
#6 Let \( G \) be the graph

Find \( \alpha(G), \beta(G), \alpha_1(G), \beta_1(G) \).

Find a maximum matching in \( G \).

#7 In the graph below find a minimal \( u-v \) separating set and a maximal set of internally disjoint \( u-v \) paths and also a minimal \( u_{10}-u_{16} \) separating set and a maximal set of internally disjoint \( u_{10}-u_{16} \) paths.
#8  State whether each of the following is true or false. If true, give a proof. If false, give a counterexample.

(a) If e is a bridge of G and v is incident to e, then v is a cut vertex.

(b) If v is a vertex of G and e is incident to v, then e is a bridge.

(c) If S is a vertex cut in a connected graph G, then G - S has exactly two components.

(d) If X is an edge cut in a connected graph G, then G - X has exactly two components.

(e) If G is a connected graph with exactly two vertices of odd degree and deg v is odd, then at most one edge incident to v is a bridge.
#9  Let $G = \text{graph}$.

Find $\overline{C(G)}$.

#10 Show that any nontrivial connected graph contains at least two vertices that are not cut vertices.

#11 Prove that a connected graph in which every vertex has degree 2 is a cycle.

#12 Let $R$ be the relation on $E(G)$ where $G$ is a nontrivial connected graph defined by $e R f$ if $e, f \in E(G)$ and either $e = f$ or $e$ and $f$ lie on a common cycle.

(a) Prove $R$ is an equivalence relation.

(b) Show that if $e R f$ then $e$ and $f$ belong to the same block of $G$. 

#13 (a) For any integers \( 1 \leq m \leq n-1 \) connected
give an example of a graph with
\( m \) cut vertices and \( n \) blocks.

(b) Prove that if \( G \) is a nontrivial
connected graph with \( m \) cut vertices
and \( n \) blocks then \( m \leq n-1 \)

#14 You should know the definitions
of:
- cut-vertex
- vertex cut, maximum vertex cut
- maximal vertex cut
- edge cut, maximum edge cut
- maximal edge cut
- \( \kappa(G), \tau(G), \sigma(G) \)
- \( \alpha(G), \chi(G), \beta(G), \beta_i(G) \)
- \( u-v \) separating set
- internally disjoint paths
- blocks
- matching
- independent set of edges