Problem Set 2

Sections numbered with a § sign and problems numbered with a # sign are identical to the textbook for this course *Elementary Number Theory and its applications* by Kenneth H. Rosen.

You are encouraged to work on these problems in groups.

§3.1 Prime Numbers

#6 Show that no integer of the form n^3+1 is a prime, other than $2 = 1^3+1$.

#12 (a) Find the smallest five consecutive composite integers.

(b) Find one million consecutive composite integers.

#14 Verify Goldbach's Conjecture for each of the following numbers.

(a) 98

(b) 102

The Riemann ζ-function

For the following 2 problems you may want to recall what you have learned about the convergence of series in Calc II.

1. Show that $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ converges for $s > 1$.
2. Show that $\zeta(1)$ diverges. (Hint: Use mathematical induction to show that $\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{2^n} > \frac{1+n}{2}$)

§3.2 Greatest Common Divisors

#2 Find the greatest common divisor of each of the following pairs of integers

c) -27, -45

e) 100, 121

#4 Let a be a positive integer. What is the greatest common divisor of a and a^2?

#8 Show that if a and b are integers with $(a, b) = 1$, then

$$(a + b, a - b) = 1 \text{ or } 2$$

#12 Show that if a, b and c are integers such that $(a, b) = 1$ and $c|(a + b)$, then $(c, a) = (c, b) = 1$.

#14 (a) Show that if a, b, and c are integers such that $(a, b) = (a, c) = 1$, then $(a, bc) = 1$.

(b) Use mathematical induction to show that if $a_1, a_2, \ldots, a_n$ are integers, and b is another integer such that $(a_1, b) = (a_2, b) = \ldots = (a_n, b) = 1$, then $(a_1 a_2 \cdots a_n, b) = 1$.

Extra Credit Problems – due Monday, 6/7/03

You may work on the following problems in a group of people as long as you state all the names of people who worked together. You will get extra credit for each correct solution. Credit will be divided by the number of people working together.

§3.1

#16 Show that every integer greater than 11 is the sum of two composite integers.

§3.2

#30 (a) Show that if a and b are positive integers, then

$$\left(\frac{a^n - b^n}{a - b}, a - b\right) = \left(n(a,b)^{n-1}, a - b\right)$$

(b) Show that if a and b are relatively prime positive integers, then

$$\left(\frac{a^n - b^n}{a - b}, a - b\right) = (n, a - b)$$