Name: Math 136 Workshop One

1. The fibonacci sequence is an increasing sequence of numbers defined by $a_1 = 1$, $a_2 = 1$, and in general $a_{n+1} = a_n + a_{n-1}$.

   (a) Read the rabbit thing in problem 8.1.48 on page 504 to see an example of fibonacci numbers in nature.

   (b) Write out the first 13 terms of the fibonacci sequence.

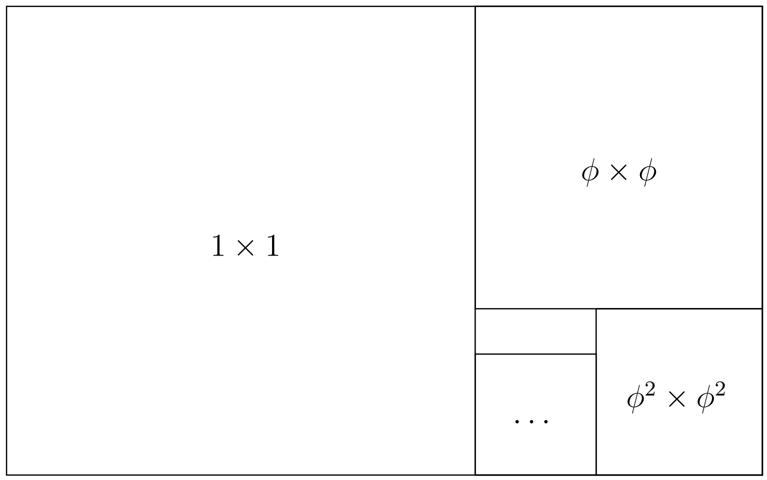
   (c) Is the fibonacci sequence geometric?

   (d) Find the limit of the a_n.

   (e) From term a_n to term a_{n+1}, the fibonacci sequence grows by a factor of $r_n = a_{n+1}/a_n$. This defines a new sequence!
   Prove that $r_{n+1} = 1 + \frac{1}{r_n}$.

   (f) By the monotone convergence theorem, the r_n converge to some limit r. We say that r_n is asymptotically geometric, with asymptotic ratio r.
   Find r.

2. Consider a rectangle tiled by shrinking squares as shown:



Imagine that the squares continue and get infinitely smaller.

(a) Explain why the area of the whole rectangle is $1 + \phi$.

(b) Explain why the area of the whole rectangle is $1 + \phi^2 + \phi^4 + \phi^6 + \cdots$.

(c) Use the geometric series formula to evaluate this infinite sum symbolically.

(d) Set (a) and (c) equal to solve for ϕ.

(e) Does your answer look similar to the previous page?