1. You all know the Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$. It is the sequence inductively defined by $a_1 = a_2 = 1$ and
$$a_{n+1} = a_n + a_{n-1}$$
   (a) Determine whether a_n converges or diverges.

   For $n \geq 2$, define the sequence of successive ratios of a_n by
$$r_n = \frac{a_n}{a_{n-1}}$$
   (b) Write the first six terms of r_n.
   (c) Find a recurrence for r_n. In particular, find a formula for r_{n+1} in terms of only r_n.
   (d) Even though r_n is not monotone, take my word for it that it has a limit. Use your answer for part (c) to find it symbolically.
   (e) Determine whether $\sum r_n$ converges or diverges.

2. Consider the sequence inductively defined by $a_1 = 1$ and
$$a_{n+1} = \sqrt{1 + a_n}$$
   (a) Write the first six terms of a_n, accurate to 3 decimal digits.
   (b) Use induction to show that this sequence is monotonically increasing.
   (c) Use induction to show that this sequence is bounded above by 10.
   Hence by the Monotone Sequence Theorem, a_n converges.
   (d) Find the limit of a_n symbolically.