

1. Consider the function $f(x) = x^5$.
 - (a) Find the first-degree Taylor polynomial of f at $a = 2$. (In other words just find the tangent line!)
 - (b) Find the second-degree Taylor polynomial of f at $a = 2$.
 - (c) If your answer in (b) is used to approximate f in the interval $[2, 2.1]$, can you be certain that the error is less than 0.05? Explain using Taylor's inequality.
2. Consider the function $f(x)$ defined by the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

- (a) What is the domain of f ? (In other words, find the interval of convergence.)
- (b) Write out several terms and verify that $f' = -2xf$. (In other words, the series for f' is the same as the series for $-2xf$.) (Hence f is just a solution to the differential equation $y' = -2xy$, with initial condition $y(0) = 1$.)
- (c) Solve the initial value problem in (b) to find an ordinary formula for f .