

1. (10 points) Write your guess for the form of the partial fraction decomposition of

$$\frac{1}{x^3 - x}$$

Do *not* solve for the constants. Do *not* integrate.

2. (10 points) Evaluate the improper integral  $\int_0^{\infty} \frac{dx}{(x+1)^2}$

Hint: partial fractions does *not* make sense here.

3. (10 points) Solve the differential equation  $y' = xe^y$  for  $y$  as a function of  $x$  subject to the condition  $y(1) = 0$ .

4. (20 points) Consider the region in the first quadrant bounded by the  $x$ -axis, the  $y$ -axis, the line  $x = 2$ , and the curve  $y = 1/(1 + x^2)$ . Find the volume of the solid obtained by rotating this region about the  $y$ -axis. Simplify your answer.

5. (20 points) A tank holds  $20g$  of pure water. A solution containing  $3lb/g$  of salt enters the tank at a rate of  $2g/m$ . The contents of the tank are mixed and drain from the tank at a rate of  $3g/m$ .
- (a) When will the tank be empty?
  - (b) When will the tank be half-empty?
  - (c) If  $S(t)$  represents the amount of salt in the tank at time  $t$ , write the differential equation that  $S$  satisfies.
  - (d) When the tank is half-empty, how much salt is in the tank? You do *not* need to simplify your answer.

6. (10 points) Suppose that the consumer's demand for some commodity is described by  $D(q) = 50 - 3q - q^2$ . Suppose that the present market quantity is  $q_0 = 3$ . Find the consumer surplus for this market. You do *not* need to simplify your answer.

7. (10 points) Evaluate the integral

$$\int \sin^{-1} x \, dx$$

Hint: use the fact that

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$