- Trigonometry of the unit circle, the relationships between all six trig functions and their inverse functions. Identities like $\cos^2 x = (1+\cos(2x))/2$, $\sin^2 x = (1-\cos(2x))/2$, $\sin^2 x+\cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$.
- Limits, L'Hôpital's rule, derivatives, chain rule, product rule.
- Basic integration laws concerning x^p, $\sin x$, $\cos x$, $\tan x$, $\ln x$, e^x, $1/(1+x^2)$, and anything else we've used in class.

6.1 The area of the region between two functions on an interval $[a, b]$ is $A = \int_a^b (\text{top} - \text{bottom})dx$. Be careful if the functions cross!

Try problems 10, 16, 18.

6.2 The volume of a solid with known x-cross-sectional areas $A(x)$ for x in an interval $[a, b]$ is given by

$$V = \int_a^b A(x)dx$$

A special case is a solid of revolution about some axis. Strips perpendicular to the axis sweep out washers, and strips parallel to the axis sweep out cylinders.

An example of the washer method: Suppose the axis of rotation is the x-axis. Then the x strips sweep out washers and so

$$V = \int_a^b (\pi R(x)^2 - \pi r(x)^2)dx$$

where $R(x)$ is the outer radius and $r(x)$ is the inner radius.

An example of the cylinder method: Suppose the axis of rotation is the y-axis. Then each vertical strip sweeps out a cylinder and so

$$V = \int_a^b 2\pi r(x)h(x)dx$$

where $r(x)$ is the radius and $h(x)$ is the height.

Warning! The axis of revolution can always be either x or y, or some other line.

Try problems 9, 14, 31c, 41ab.

6.6 The key terms are p = price, q = quantity, and $p = D(q)$ is the demand function. If the market has q_0 items then the item price is $D(q_0)$ so the revenue is $R = \text{price} \times \text{quantity} = q_0 D(q_0)$. The consumer's surplus for this market is

$$CS = \int_0^{q_0} D(q)dq - q_0 D(q_0)$$

Similarly, $p = S(q)$ is the supply function. The optimal market is at the q_0 where $D(q_0) = S(q_0)$.

I don't really care about producer's surplus.

Try problems 18, 26.

7.1 The substitution rule is, in short,

$$\int g(u)u'dx = \int g(u)du$$

Here's what this means. Suppose $\int f(x)dx$ is an integral that you don't know a formula for. If you can find some $u(x)$ and write the integral in the form $\int g(u)u'dx$, then you can replace the $u'dx$ with du and try to solve $\int g(u)du$ instead.

Try problems 6, 10, 46.

7.2 The integration by parts rule is, in short

$$\int uv'dx = uv - \int u'vdx$$

The $u(x)$ and $v'(x)$ are your choice.

Try problems 8, 9, 10, 11.

7.3 How do you integrate $\int sin^n(x)cos^m(x)dx$? There are two cases.

Try $\int \sin^3 x \cos^2 x dx$, $\int \sin^4 x \cos^3 x dx$, $\int \sin^2 x \cos^2 x dx$.

7.4 Here is the partial fraction method: Long divide if necessary, make your guess for the decomposition, get a common denominator and equate numerators, collect terms and equate coefficients, solve for the constants.

The main thing is to remember how to make the guess for the decomposition. Remember that there should be as many constants as the degree of the denominator. Here is an example guess:

$$\frac{1}{(x-1)^3(x^2+1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

But of course this one is too big to solve by hand.

Try problems 16, 28, 32, 12.

You can practice methods of integration using the problems in section 7.5. But some of those problems use trigonometric substitution, which we didn't learn.

5.6 You should be able to check whether a given function y is a solution to a given differential equation.

A differential equation is separable if it can be written $Q(y)y' = P(x)$. Here is the method for solving these: integrate both sides dx, substitute $y'dx \to dy$ on the left, integrate, solve for the constant if you can, isolate y if asked.

The exponential growth model is $Q' = kQ$ and has solution $Q(t) = Q_0 e^{kt}$.

Try problems 6, 27, 51.

7.6 A differential equation is first-order linear if it can be written $y' + P(x)y = Q(x)$. The general solution is

$$\frac{1}{I}(\int IQ + C)$$

where $I = e^{\int P}$ is the integrating factor.

The main application is in-flow/out-flow problems, where we use the general rule $Q' =$ (rate in) - (rate out).

Try problems 6, 7, 23, 34.

7.7 I will not test you on the whole section. You should know that $\int_1^\infty f(x)dx$ means to integrate as normal and then take a limit. For example you can easily find $\int_1^\infty \frac{1}{x^p}dx$ for any p.

Try problems 3, 5, 13, 17.