1. (a) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$$

Show that A **cannot be** diagonalized.

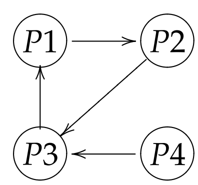
(b) Suppose now that A is some other 3×3 matrix which has exactly 3 distinct eigenvalues. Explain briefly but clearly why A **must be** diagonalizable.

2. In a certain country, there are three political parties: The conservative Butter-side-up party (the Uppers), the radical Butter-side-down party (the Downers), and the progressive Pita-pocket party (the Pocketeers). Recent polls show that each year, 70% of Uppers remain Uppers, 80% of Downers remain Downers, 40% of Pocketeers remain Pocketeers. Everyone else switches, and they divide equally among the other two parties.

(a) Write the transition matrix representing the party-switching process.

(b) In the long run, how will the people be distributed among the political parties?

3. Consider the following very small internet.



Assume that surfers have an 80% chance of following one of the links on the page, and a 20% chance of jumping to a random page.

(a) Write the transition matrix A representing the surfing process.

(b) Is A singular or nonsingular?

4. **CHOOSE ONLY ONE** of the following problems.

(1c) Let A and B be unkown diagonalizable matrices. If A and B have exactly the same eigenvectors as each other, then show that $AB = BA$.

(2c) In problem 2, if initialy everybody is an Upper, then use diagonalization to write out a formula in terms of n for the distribution after n years.

(3c) In problem 3, what are the Page Rankings for this internet?