

1. Let  $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}$ , and  $C = \begin{bmatrix} 1 & 8 \\ -1 & 4 \end{bmatrix}$ .

(a) Use row reduction to find  $A^{-1}$ .

(b) Use your previous answer to solve the matrix equation  $XA + B = C$ .

2. Let  $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ .

(a) Compute the REF of  $A$ .

(b) What is the determinant of  $A$ ?

(c) Find a basis for the column space of  $A$ .

(d) Find a basis for the row space of  $A$ .

(e) What is the nullity of  $A$ ?

3. Let  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$  be vectors in  $\mathbb{R}^3$ . Answer the following questions, and include a one sentence explanation of your response.

(a) Can you tell if they are linearly independent?

(b) Can you tell if they span  $\mathbb{R}^3$ ?

(c) What can you say about the rank of the matrix  $V = \begin{bmatrix} | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \\ | & | & | & | \end{bmatrix}$ ?

(d) What can you say about the nullity of  $V$ ?

4. Consider the basis  $\mathcal{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  for  $\mathbb{R}^2$ .
- (a) What is the transition matrix from  $\mathcal{B}$  to the standard basis  $\mathcal{E}$ ? Use it to write  $\mathbf{v}_{\mathcal{B}} = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$  in standard coordinates.
- (b) What is the transition matrix from  $\mathcal{E}$  to  $\mathcal{B}$ ? Use it to write  $\mathbf{w} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$  in  $\mathcal{B}$ -coordinates.
- (c) Write the matrix  $A = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$  relative to the basis  $\mathcal{B}$ .