FINAL EXAM PRACTICE PROBLEMS

(1) Compute $\int_{\mathcal{C}} ds$, where $\mathcal{C}$ is the unit circle.

(2) Compute $\int_{\mathcal{C}} ds$, where $\mathcal{C}$ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(3) For what (real number) value of α is the line integral

$$\int_{\mathcal{C}} (xy^2 + \alpha x^2 y)\, dx \;+\; (x^3 + x^2 y)\, dy$$

independent of path?

(4) Use Stokes' Theorem to compute the integral $\iint_{\mathcal{S}} \operatorname{curl} \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle xz,\, yz,\, xy\rangle$ and $\mathcal{S}$ is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

(5) Compute $\int_{\mathcal{C}} xy^4\, ds$, where $\mathcal{C}$ is the right half of the circle $x^2 + y^2 = 4$.

(6) Find a vector field $\vec{F}$ such that curl $\vec{F} = \langle xy^2,\, yz^2,\, zx^2\rangle$, or, if this is not possible, explain why.

(7) Find the area of the region enclosed by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 4$.

(8) Evaluate the surface integral $\iint_{\mathcal{S}} \langle y^2,\, 2,\, -x\rangle \cdot d\vec{S}$, where $\mathcal{S}$ is the portion of the plane $x+y+z = 1$ in the octant $x, y, z \geq 0$, oriented with normal vector pointing upwards.

(9) Compute $\int_{\mathcal{C}} (y + e^{\sqrt{x}})\, dx \;+\; (2x + \cos y^2)\, dy$, where $\mathcal{C}$ is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

(10) Let S be the surface defined by the equation $x^2 + y^2 - z^2 = 2x(y + z) - 2$.
a) Find the equation of the plane tangent to S at the point $P = (1, 0, 1)$.
b) The equation for S defines z implicitly as a function of x and y.
Compute $\partial z/\partial x$.

(11) Evaluate the surface integral $\iint_{\mathcal{S}} e^{-z}\, dS$, where $\mathcal{S}$ is the cylinder $x^2 + y^2 = 4$, $0 \leq z \leq 4$.

(12) Find all local and absolute extreme values of the function $f(x, y) = 4x^2 - 4xy + y^2$ on the closed disc of radius 5 centered at the origin.

(13) Find the work done by the force field $F = \langle e^x \cos y + yz, xz - e^x \sin y, xy + z\rangle$ over the line segment from the origin to $(1, \pi, 2)$.

(14) Use Gauss' Theorem to calculate the flux of the vector field $\vec{F}(x, y, z) = \langle e^x \sin y,\, e^x \cos y,\, yz^2\rangle$ across the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$.

(15) Find $\int_{\mathcal{C}} (2xy + \cos x)\, dx \;+\; x^2\, dy$, where $\mathcal{C}$ is the upper half of the ellipse $x^2/4 + y^2 = 1$, traversed counter-clockwise.

(16) Find the flux of $\vec{F}$ across the surface $\mathcal{S}$, where $\vec{F}(x,y,z) = \langle x^2y^3,\, e^x \sin z,\, \sqrt{z}\rangle$, and $\mathcal{S}$ is the intersection of the solid elliptical cylinder $\{(x,y,z) \mid y^2 + \frac{x^2}{4} \leq 1\}$ with the xy-plane.

(17) Compute $\iint_D (2y - \frac{x}{2})\, dA$, where D is the region $\{(x,y) \mid \frac{x^2}{4} + y^2 \leq 1\}$.

(18) Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{s}$, where $\vec{F}(x,y,z) = \langle \sin x,\, \cos y,\, xz\rangle$ and $\mathcal{C}$ is the curve parameterized by $\vec{r}(t) = \langle t^3, -t^2, t\rangle$, $0 \leq t \leq 1$.

(19) Determine whether or not the following vector fields are conservative. If conservative, find a potential function; if not, compute the curl.
a) $\vec{F}(x,y) = \langle ye^x + \sin y,\, e^x + x\cos y\rangle$
b) $\vec{F}(x,y,z) = \langle xz,\, xyz,\, -y^2\rangle$
c) $\vec{F}(x,y,z) = \text{curl}\, \nabla f$, where f is a smooth scalar field on $\mathbb{R}^3$.

(20) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = a^2$ that lies inside the cylinder $x^2 + y^2 = ax$.

(21) Use Stokes' Theorem to evaluate $\int_{\mathcal{C}} x + y^2\, dx \; + y + z^2\, dy \; + z + x^2\, dz$, where $\mathcal{C}$ is the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

(22) Find a parametric representation for the torus obtained by rotating the circle $(y-2)^2 + z^2 = 1$ about the z-axis.

(23) Use Green's Theorem to compute

$$\int_{\mathcal{C}} x^2y^2\, dx + 4xy^3\, dy,$$

where $\mathcal{C}$ is the triangle with vertices $(0,0)$, $(1,3)$, and $(0,3)$.

(24) Find the work done by the force field $\vec{F}(x,y) = \langle x\cos y,\, \sec^2 y\rangle$ on a particle that moves along the parabola $y = x^2$ from the origin to $(\frac{\sqrt{\pi}}{2}, \frac{\pi}{4})$.

(25) Evaluate

$$\int_{\mathcal{C}} \sin y\, dx + x\cos y\, dy,$$

where $\mathcal{C}$ is the ellipse $x^2 + xy + y^2 = 1$.

(26) Determine whether or not the vector field $\vec{F}(x,y) = \langle x^3 + 4xy,\, 2x^2 - y^2 + 1\rangle$ is conservative. If it is, find a potential function for $\vec{F}$ — ie, a scalar field f such that $\vec{F} = \nabla f$.

(27) Evaluate $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$, where $\vec{F}(x,y,z) = \langle 4x^3z,\, 4y^3z,\, 3z^4\rangle$ and $\mathcal{S}$ is the sphere of radius R centered at the origin.

(28) How much time does it take a particle whose motion is given by the space curve $\mathbf{r}(t) = \langle \sin t,\, \cos t,\, \sqrt{8}t\rangle$ to move a distance of 6 units, starting from the point $(0,1,0)$?

(29) Calculate the directional derivative of the function $f(x,y) = 5x + 2x\sqrt{y}$ in the direction $\langle 4,3\rangle$ at the point $(75,25)$.

(30) Find the absolute maximum value of the function $f(x,y) = 30x^2y^2 + 30xy^3 - 120xy^2 + 60$ on the region D, where D is the triangle with vertices $(0,0)$, $(6,0)$, and $(0,6)$, including the boundary.