PRACTICE PROBLEMS FOR SECOND MIDTERM

(1) Find the directional derivative of the function $f(x, y, z) = \dfrac{x}{y + z}$ at the point $(4, 1, 1)$ in the direction $\langle 1, 2, 3\rangle$.

(2) Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$, where $z = \arctan(2x + y)$, $x = s^2 t$, and $y = s \ln t$.

(3) Find the maximum rate of change of $z = f(x, y) = \sin(xy)$ at the point $(1, 0)$ and the direction in which it occurs.

(4) Find all local maximum and minimum values and saddle points of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

(5) Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2 y + 4$ on the set $D = \{(x, y) : |x| \leq 1, |y| \leq 1\}$.

(6) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = 2x + 6y + 10z$, subject to the constraint $x^2 + y^2 + z^2 = 35$.

(7) Evaluate
$$\int_0^3 \int_{y^2}^9 y \cos(x^2)\, dx\, dy$$

(8) Find the volume of the solid in the first octant bounded by the cylinder $z = 9 - y^2$ and the plane $x = 2$.

(9) Classify the surfaces $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$, $r = 3$, and $\phi = \pi/4$.

(10) Evaluate
$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2}\, dz\, dy\, dx$$

(11) Evaluate
$$\iint_R \arctan(y/x)\, dA, \text{ where } R = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}.$$

(12) Evaluate
$$\iiint_E y^2 z^2\, dV,$$
where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.