WORKSHOP 1: DUE TUESDAY, JULY 7TH

Work through the following four problems together with the other members of your group, in the class time provided for you. Use books, notes, or any resource you can think of. Then write up formal solutions to the problems assigned at the end of class, and submit your answers next Tuesday.

(1) Reparametrize the curve

$$\vec{r}(t) = \left\langle \frac{2}{t^2+1} - 1,\ \frac{2t}{t^2+1} \right\rangle, \quad 0 \leq t < \infty$$

with respect to arc length measured from the point $t = 0$ in the direction of increasing t, making sure to give the correct interval over which the new parameter varies. Describe in words the path traversed through the plane by a particle whose motion is given by $\vec{r}(t)$, indicating both the shape of the path and the rate of progress of the particle along the path.

(2) Let $\mathcal{P}$ be a plane in $\mathbb{R}^3$, and let Q be a point in $\mathbb{R}^3$ that does not lie in $\mathcal{P}$. Define the **distance** from $\mathcal{P}$ to Q to be the length of the line segment that is perpendicular to $\mathcal{P}$ with initial point Q and terminal point lying in $\mathcal{P}$. Derive a general formula for the distance between an arbitrary plane $ax + by + cz = d$ and an arbitrary point (x_0, y_0, z_0), both in $\mathbb{R}^3$. Then, use your formula to find the distance between the plane $3x - 4y + 12z = 21$ and the point $(2, -1, 2)$.

(3) Let E be the ellipse in $\mathbb{R}^3$ with vertices $(1, -1, -2)$, $(-2, 2, 2)$, $(1, 5, 6)$, and $(4, 2, 2)$. Find the center and the foci of E, and find the lengths of the major and minor axes of E. Finally, find a parametrization of E.

(4) Recall the Mean Value Theorem from Calc 1:

> If $f : \mathbb{R} \to \mathbb{R}$ is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then there exists a point $c \in (a, b)$ such that
> $$f'(c)(b - a) = f(b) - f(a).$$

Does the Mean Value Theorem extend to vector-valued functions? That is, consider the following possible theorem:

> If $\vec{r} : \mathbb{R} \to \mathbb{R}^3$ is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then there exists a point $c \in (a, b)$ such that
> $$\vec{r}\,'(c)(b - a) = \vec{r}(b) - \vec{r}(a).$$

Determine whether or not this "theorem" is true, and if it is not, provide a counter-example.