

ANSWERS TO EXAM #2

2

1. If $g(x) = \ln(1 + x^2)$, find $g'(x)$.

$$\boxed{g'(x) = \frac{2x}{1+x^2}}$$

2. Find the equation of the tangent line to the curve $x^2y^2 + 2xy^3 = 20$ at $(1,2)$.

$$2x^2y^2 + 2x^2y\ y' + 2y^3 + 2x \cdot 3y^2y' = 0$$

$\Downarrow x=1, y=2$

$$8 + 4y' + 16 + 24y' = 0$$

$$28y' = -24$$

$$y' = -\frac{24}{28} = -\frac{6}{7}$$

$$\boxed{y - 2 = -\frac{6}{7}(x - 1)}$$

3.

a. Find $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 7}{3x^3 + 4x^2 + 2}$. Be sure to show your work.

b. Find $\lim_{x \rightarrow \infty} \frac{2 + e^{-x}}{3 + e^{-x}}$

a. $\lim_{x \rightarrow \infty} \frac{1 + \frac{2}{x^2} + \frac{7}{x^3}}{3 + \frac{4}{x} + \frac{2}{x^3}} = \frac{1+0+0}{3+0+0} = \boxed{\frac{1}{3}}$

b. $\lim_{x \rightarrow \infty} \frac{2 + e^{-x}}{3 + e^{-x}} = \frac{2+0}{3+0} = \boxed{\frac{2}{3}}$

4. If $f(x) = x + \frac{1}{x}$ find the c guaranteed by the mean value theorem on the interval $[2, 8]$.

$$f'(x) = 1 - \frac{1}{x^2} \quad \frac{f(8) - f(2)}{8 - 2} = \frac{\frac{45}{8}}{6} = \frac{45}{48} = \frac{15}{16}$$

$$1 - \frac{1}{c^2} = \frac{15}{16}$$

$$-\frac{1}{c^2} = \frac{15}{16} - 1 = -\frac{1}{16}$$

$$c^2 = 16 \quad c = \pm 4$$

$c = +4$ is in $[2, 8]$

5. Find the absolute maximum and minimum of the function $f(x) = (x-1)^2(x-4)$ on the interval $[0, 5]$.

Absolute maximum:	$16 \text{ at } x=5$
Absolute minimum:	$-4 \text{ at } x=0, 3$

$$\begin{aligned}f'(x) &= 2(x-1)(x-4) + (x-1)^2 \cdot (1) \\&= (x-1)[2(x-4) + (x-1)] = (x-1)(3x-9)\end{aligned}$$

$$f'(x) = 0 \text{ when } x=1, x=9$$

$$f(0) = -4$$

$$f(1) = 0$$

$$f(3) = -4$$

$$f(5) = 16$$

6. Use either linear approximation or differentials to estimate 1.01^9 .

$$f(x) \approx f(a) + f'(a)(x-a) \quad f(x) = x^9 \quad a=1$$

$$f'(x) = 9x^8$$

$$f(1.01) \approx 1^9 + 9 \cdot 1^8(1.01-1) = 1 + 9(.01) = 1.09$$

$1.01^9 \approx 1.09$

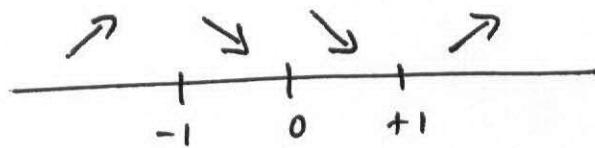
7. Find the intervals where the function $f(x) = x + \frac{1}{x}$ is increasing and decreasing. Find all relative maxima and minima and horizontal and vertical asymptotes of this function.

Intervals where increasing:	$(-\infty, -1) \cup (1, \infty)$
Intervals where decreasing:	$(-1, 0) \cup (0, 1)$
Horizontal asymptotes:	None
Vertical asymptotes:	$x = 0$
Relative maxima:	$x = -1, f(-1) = -2$
Relative minima:	$x = 1, f(1) = 2$

$$f'(x) = 1 - \frac{1}{x^2}$$

$$1 - \frac{1}{x^2} = 0 \Leftrightarrow x = \pm 1$$

$f'(x)$ undefined for $x = 0$



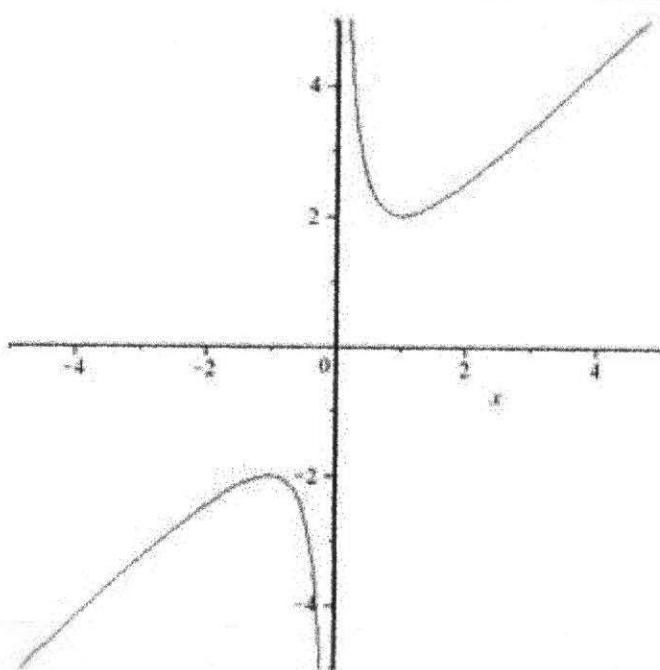
$x = 0$ is VA

$x = -1, f(-1) = -2$ is rel max

$x = 1, f(1) = 2$ is rel min

$$\begin{aligned}\lim_{x \rightarrow \infty} f(x) &= \infty \\ \lim_{x \rightarrow -\infty} f(x) &= -\infty\end{aligned}$$

No
HA's

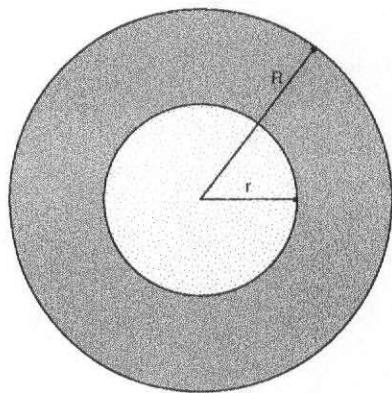


I've included a sketch for completeness. This is not required for the problem.

8. The area of a ring with inner radius r and outer radius R is

$$A = \pi(R^2 - r^2).$$

If, at a certain time, $R = 2$, $r = 1$, R is decreasing by $1/2$ ft/min, and r is increasing by 1 ft/min, how fast is the area of the ring changing at that time?

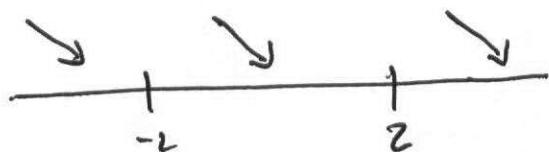
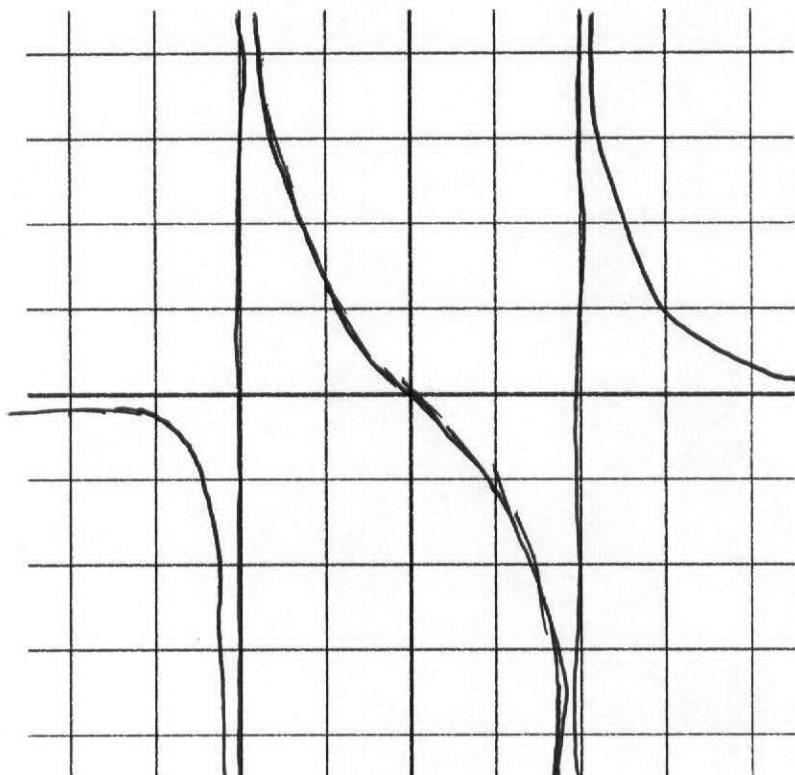


$$\begin{aligned}\frac{dA}{dt} &= \pi \left(2R \frac{dR}{dt} - 2r \frac{dr}{dt} \right) \\ &= \pi \left(2 \cdot 2 \left(-\frac{1}{2} \right) - 2 \cdot 1 \cdot 1 \right) \\ &= \pi (4(-\frac{1}{2}) - 2) = -4\pi\end{aligned}$$

$$\text{So } \frac{dA}{dt} = -4\pi \text{ ft}^2/\text{min}$$

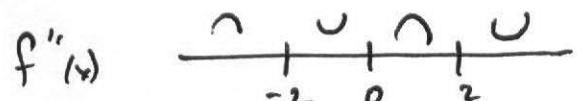
9. Let $f(x) = \frac{x}{x^2 - 4}$. For this function, $f'(x) = -\frac{x^2 + 12}{(x^2 - 4)^2}$ and $f''(x) = \frac{2x(x^2 + 12)}{(x^2 - 4)^3}$. Use this information to sketch the graph of $y = f(x)$.

Intervals where increasing:	<u>NONE</u>
Intervals where decreasing:	$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
Intervals where concave up:	$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
Intervals where concave down:	$(-\infty, -2) \cup (0, 2)$
Horizontal asymptotes:	$y = 0$
Vertical asymptotes:	$x = \pm 2$
Inflections:	$y = 0$



$$\underline{f'(x)}$$

$$\lim_{x \rightarrow \pm\infty} f'(x) = 0$$



$x = \pm 2$ are not inflection points because $f(x)$ is not defined at those points

10. If $y = x^{7x^2}$, find $\frac{dy}{dx}$.

$$\ln y = 7x^2 \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 14x \ln x + 7x^2 \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y(14x \ln x + 7x)$$

$$\boxed{\frac{dy}{dx} = x^{7x^2}(14x \ln x + 7x)}$$