EXAM I, FALL 2013, ANSWERS

1. Find the equations of the tangent and normal lines to the graph of $y = (2x + x^3)$ at x=1. (Recall: The normal line is perpendicular to the tangent line.)

Tangent line:	$y - 3 = 5(x-1)$
Normal line:	$y - 3 = -\frac{1}{5}(x-1)$

$y' = 2+3x^2$ at $x=1$, $y' = 5$.
$y = 3$

2. Since the beginning of the month, the hot water heater in George's garage has been losing water at a constant rate (that is, the amount of water in the water heater is a linear function of time). On the 12th of the month, the water heater held 30 gallons of water; on the 21st, it held only 15 gallons. How much water was in the hot water heater on the 8th of the month?

Gallons of water:	$\frac{110}{3}$

w	t
30	12
15	21

$$(w - 30) = \frac{\Delta w}{\Delta t}(t-12)$$

$$= -\frac{15}{9}(t-12)$$

$$w = -\frac{5}{3}t + 20 + 30 = -\frac{5}{3}t + 50$$

$$t = 8 \Rightarrow w = -\frac{5}{3}\cdot 8 + 50 = \frac{110}{3}$$

THE PROBLEM TELLS YOU THAT W IS A LINEAR FUNCTION OF TIME, SO YOU SHOULD NOT USE THE FORMULA FOR EXPONENTIAL GROWTH/DECAY.

3. Show that the equation $\sqrt[3]{x} = x^2 + 2x - 1$ has at least one solution on the interval (0, 1). Justify your answer.

(I) $f(x) = \sqrt[3]{x} - x^2 - 2x + 1$

(II) $f(x)$ is continuous because it is the sum of a polynomial and a power function.

(III) $f(0) = 1$, $f(1) = -1$, so by IRLT or IVT, there is a c in $(0,1)$ so that $f(c) = 0$.

This says that $\sqrt[3]{c} = c^2 + 2c - 1$

You need to give f(x) a name and say why it's continuous.

4. Find the derivatives. You do not have to simplify your answers once you have finished differentiating.

a. $f(x) = \dfrac{2}{3x^2} + \dfrac{x}{3} + \dfrac{4}{5} + \dfrac{x+1}{x}$.

$= \frac{2}{3}x^{-2} + \frac{1}{3}x + \frac{4}{5} + 1 + \frac{1}{x}$

$f'(x) = \left(\frac{2}{3}\right)(-2)x^{-3} + \frac{1}{3} + 0 + 0 - \frac{1}{x^2}$

$f'(x) = -\frac{4}{3}x^{-3} + \frac{1}{3} - \frac{1}{x^2}$

b. $g(x) = \dfrac{e^x}{\sin x}$.

$g'(x) = \dfrac{(\sin x)e^x - e^x(+\cos x)}{\sin^2 x}$

5. If a bacterial colony starts with a population of 100 and takes 7 days to triple, how long will it take to quintuple?

Time equals:	$\frac{7\ln 5}{\ln 3}$

$P(t) = 100e^{kt}$

$300 = 100e^{7k}$

$3 = e^{7k}$

$\ln 3 = 7k$

$k = \frac{\ln 3}{7}$

$P(t) = 100e^{\frac{\ln 3}{7}t}$

$500 = 100e^{\frac{\ln 3}{7}t}$

$5 = e^{\frac{\ln 3}{7}t}$

$\ln 5 = \frac{\ln 3}{7}t$

$t = \frac{7\ln 5}{\ln 3}$

6. a. If $f(x) = x^5 h(x)$, $h(-1) = 4$, and $h'(-1) = -7$, find $f'(-1)$.

$f'(x) = 5x^4 h(x) + x^5 \cdot h'(x)$

$f'(-1) = 5(-1)^4 h(-1) + (-1)^5 h'(-1)$

$= 5 \cdot 4 + (-1)(-7) = 27$

b. Find the derivative of $g(x) = \sin(e^x)$.

$g'(x) = \cos(e^x) \cdot e^x = e^x \cos e^x$

7. Find a so that

$$f(x) = \begin{cases} \frac{\sqrt{x}-1}{x-1}, & \text{if } x > 1 \\ a & \text{if } x \leq 1 \end{cases}$$

is continuous for all x.

a=	$\frac{1}{2}$

$$a = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{x-1}} \cdot \frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

8. a. Find the solutions to $\ln(x-1) + \ln(x) = \ln(4x-6)$.

$$\ln x(x-1) = \ln(4x-6)$$

$$x(x-1) = 4x-6$$

$$x^2 - x = 4x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x = 2, 3$$

b. Find $\lim_{x\to 1} \frac{\frac{1}{x}-1}{x}$. $= \frac{1-1}{1} = \frac{0}{1} = 0$

9. A person standing on the edge of a cliff throws a rock directly upward. It is observed that 2 seconds later the rock is at its maximum height (in ft) and that 3 seconds after that, it hits the ground at the base of the cliff. How high is the cliff? With what velocity does the rock hit the ground?($h(t) = -\frac{1}{2}gt^2 + v_0t + s_0$, and g=32 ft/sec^2 near the earth's surface.)

Cliff's height is:	80

Impact velocity is:	-96

$h(t) = -16t^2 + v_0 t + S_0$

$v(t) = -32t + v_0$

$v(2) = 0 \Rightarrow -32 \cdot 2 + v_0 = 0 \Rightarrow v_0 = 64$

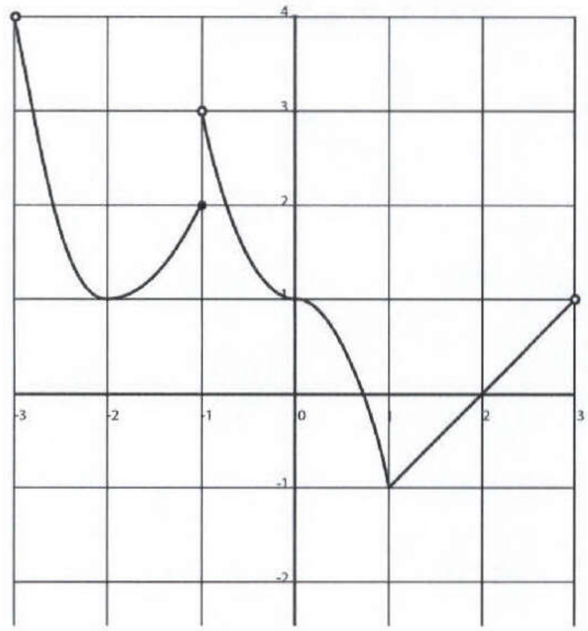
$h(t) = -16t^2 + 64t + S_0$

$h(5) = 0 \Rightarrow -16 \cdot 25 + 64 \cdot 5 + S_0 = 0$

$S_0 = 400 - 320 = 80$

$v(5) = -32 \cdot 5 + 64 = -160 + 64 = -96$

10. The graph of a function $f(x)$ is given below



Find all values of x where f fails to be

a. continuous.

$x = -1$

b. differentiable.

$x = -1, 1$

c. For which values of x is the derivative of f equal to 0?

$x = -2, 0$

d. Find $\lim_{x \to -1^-} f(x)$ and $\lim_{x \to -1^+} f(x)$.

$$\lim_{x \to -1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = 3$$