Math 152, Summer 2010, Review Problems for Midterm #1

1. Find the following indefinite integrals.

a) $\displaystyle\int x^2 e^{-x}\,dx$ b) $\displaystyle\int x^2 \arctan x\,dx$ c) $\displaystyle\int \cos^4 2x \sin^3 2x\,dx$

d) $\displaystyle\int \frac{x^2 - x + 3}{(x+1)(x^2+4)}\,dx$ e) $\displaystyle\int \frac{dx}{x\sqrt{16 - x^2}}$ f) $\displaystyle\int \sin 5x \sin 7x\,dx$

2. The region R is bounded by the curves $y = 2x$, $y = \sqrt{8 + 2x^2}$ and the y-axis. Find the volume of the solid obtained by rotating R: (a) about the x-axis; (b) about the y-axis.

3. The base of a solid is the ellipse $x^2 + 4y^2 = 4$. Each cross-section of the solid perpendicular to the x-axis is a square. What is the volume of the solid?

4. Consider the curve $y = \cos x$, $0 \le x \le \pi/2$. Set up integrals representing (a) the length of this curve; (b) the area of the surface formed when the curve is revolved about the x-axis. Evaluate one of these integrals. (The other one can be approximated by numerical methods, but you're not asked to do so.)

5. Find the average value A of $\tan^2 x$ on the interval $0 \le x \le \pi/4$. Is A larger or smaller than the average of the max and min values of $\tan^2 x$ on the interval? Draw a picture which explains this.

6. Let $I = \displaystyle\int_0^2 e^{-x^2}\,dx$.

a) Using the Trapezoidal Rule with $n = 4$ subdivisions gives what approximate value for this integral? (Give an exact answer in terms of e as well as a decimal approximation.)

b) Use the error estimate for the Trapezoidal Rule to estimate the accuracy of the approximation in a).

c) Let T_N be the approximation to I obtained from the Trapezoidal Rule with N subdivisions. Find a value of N so that $|I - T_N| < 10^{-8}$.

7. In a short paragraph, explain what the resolutive-compositive process of integral calculus is all about, and what are its four steps.

8. Compute the work required to pump all the water out of the outlet at the tip of a tank in the shape of a circular cone with height h and base radius r that's filled with water. Take the mass density of water to be ρ and the acceleration due to gravity to be g.

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