CALCULUS REVIEW PROBLEMS FOR MATH 477

Calc I, II and III are prerequisites for this course in probability. To help review the essentials of those courses, be sure that you can do the following problems. Note that you absolutely must be able to compute double integrals, especially over non-rectangular and infinite regions.

1. Find the value of
\[(2/3) + (2/3)^2 + (2/3)^3 + ...\]

2. Compute the following integrals:
\[\int x^2 e^{x/3} \, dx\]
\[\int_0^\infty e^{-2x} \, dx\]
\[\int_0^\infty \int_0^\infty xe^{-x^2/2} \, dx\]

How can you determine that this integral equals 0, without any computation?

\[\int \int_D x + y \, dxdy\]
where \(D\) is the region in the first quadrant enclosed by the x-axis, \(x = 1\), and \(y = 2x\). How do you know, prior to any computation, that the value of this double integral must be between 0 and 3? Hint: Draw the region over which you are integrating.

3. What is the Taylor series, centered at 0, for \(e^x\)? For \(e^{-x}\)? For \(e^{-x^2/2}\)?

4. Rewrite the following double integral with the order of integration reversed:
\[\int_0^\infty \int_y^\infty f(x, y) \, dxdy\]
i.e. rewrite it as an integral \(dydx\) instead of \(dxdy\).
Hint: Draw the region over which you are integrating.

5. Compute \(\frac{\partial f}{\partial x}\) and \(\frac{\partial f}{\partial y}\) for
\[f(x, y) = \frac{e^{-x/y}e^{-y}}{y}\]

6. Describe the graph of the function \(z = 2 - x - 2y\), i.e. what sort of geometric object is it, and where does it intersect the coordinate axes?

7. Is "Euler" pronounced "Yooler" or "Oiler"?