4.4 hw: #1-9 (odd) from sec 4.2

Put in $A\cos(\omega t + \phi)$ form

1) $y'' + 3y' + 2y = \cos t$

$\lambda^2 + 3\lambda + 2 = (\lambda+2)(\lambda+1) \qquad \lambda = -2, -1$

$y_h = K_1 e^{-2t} + K_2 e^{-t}$

$y'' + 3y' + 2y = e^{it}$ $\cos t$ is the real part of e^{it}

$-Ke^{it} + 3iKe^{it} + 2Ke^{it} = e^{it}$

$K(-1 + 3i + 2) = 1$

$K(3i + 1) = 1 \qquad K = \frac{1}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{1-3i}{10} = \frac{1}{10} - \frac{3}{10}i$

$y_c = \left(\frac{1}{10} - \frac{3}{10}i\right)e^{it} \longrightarrow$ convert to polar form

$r = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{3}{10}\right)^2} = \sqrt{\frac{1}{100} + \frac{9}{100}} = \sqrt{\frac{10}{100}} = \frac{\sqrt{10}}{10}$

$\tan \phi = \frac{-3}{10} \cdot \frac{10}{1} = -3 \qquad \phi = -72^\circ$

$\frac{\sqrt{10}}{10} e^{i(-72^\circ)} = \left(\frac{1}{10} - \frac{3}{10}i\right)$

$y_c = \frac{\sqrt{10}}{10} e^{(-72^\circ)i} e^{it} = \frac{\sqrt{10}}{10} e^{i(-72+t)}$

$\boxed{\frac{\sqrt{10}}{10} \cos(t - 72^\circ)}$ real part

3) $y'' + 3y' + 2y = \sin t$

$\lambda^2 + 3\lambda + 2 = 0$

$y_h = K_1 e^{-2t} + K_2 e^{-t}$

$y'' + 3y' + 2y = e^{it}$ (imaginary part)

try ke^{it}

$-k\cancel{e^{it}} + 3ik\cancel{e^{it}} + 2k\cancel{e^{it}} = \cancel{e^{it}}$

$K(-1 + 3i + 2) = 1$ $\quad K = \frac{1}{3i+1} = \frac{1}{10} - \frac{3}{10}i$

same as #1

$\frac{\sqrt{10}}{10} e^{i(-72)} = \frac{1}{10} - \frac{3}{10}i$

$y_c = \left(\frac{\sqrt{10}}{10} e^{(-72)i}\right) e^{it}$

$$\boxed{\frac{\sqrt{10}}{10} \sin\left(t + (-72^\circ)\right)}$$ ← Imaginary part

5) $y'' + 6y' + 8y = \cos t$

$\lambda^2 + 6\lambda + 8 = 0 \qquad (\lambda+4)(\lambda+2) = 0$

$\lambda = -4, -2$

$K_1 e^{-4t} + K_2 e^{-2t} = y_h$

$y'' + 6y + 8y = e^{it}$ (real part)

try $y = Ke^{it}$

$-K\cancel{e^{it}} + 6iK\cancel{e^{it}} + 8K\cancel{e^{it}} = \cancel{e^{it}}$

$K(-1 + 6i + 8) = 1 \qquad K(6i+7) = 1 \qquad K = \frac{1}{6i+7} \cdot \frac{6i-7}{6i-7} = \frac{6i-7}{-85}$

$K = \frac{7}{85} - \frac{6}{85}i$

$y_c = \left(\frac{7}{85} - \frac{6}{85}i\right)e^{it}$ (convert to polar form)

$r = \sqrt{\left(\frac{7}{85}\right)^2 + \left(\frac{6}{85}\right)^2} = \sqrt{\frac{49}{7225} + \frac{36}{7225}} = \frac{\sqrt{85}}{85}$

$\tan \phi = \left(\frac{-6}{85}\right)\left(\frac{85}{7}\right) = -\frac{6}{7} \qquad \phi = -41^\circ$

$\frac{\sqrt{85}}{85} e^{(-41^\circ)i} = \frac{7}{85} - \frac{6}{85}i$

$y_c = \left(\frac{7}{85} - \frac{6}{85}i\right)e^{it} = \frac{\sqrt{85}}{85} e^{(-41^\circ)i} e^{it} = \frac{\sqrt{85}}{85} e^{i(-41^\circ + t)}$

Take the real part of y_c to get y_p

$$\boxed{y_p = \frac{\sqrt{85}}{85} \cos(t - 41^\circ)}$$

(7) $y'' + 4y' + 13y = 3\cos 2t$

$\lambda^2 + 4\lambda + 13 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16-52}}{2} = \dfrac{-4 \pm 6i}{2} = -2 \pm 3i$

$C_1 e^{-2t}\cos 3t + C_2 e^{-2}\sin 3t = y_h$

$y'' + 4y' + 13y = e^{2ti}$ $\left(\text{try } y = Ke^{2ti}\right)$

$(Ke^{2ti})'' + 4(Ke^{2ti})' + 13(Ke^{2ti}) = e^{2ti}$

$-2\,k\cancel{e^{2ti}} + 8ik\cancel{e^{2ti}} + 13k\cancel{e^{2ti}} = \cancel{e^{2ti}}$

$K(-2 + 8i + 13) = 1$

$K(11 + 8i) = 1$ $\quad K = \frac{1}{11+8i} \cdot \frac{11-8i}{11-8i} = \frac{11-8i}{121+64} = \frac{11}{185} - \frac{8}{185}i$

$y_c = \left(\frac{11}{185} - \frac{8}{185}i\right)e^{it}$ write y_c in polar form

$r = \sqrt{\left(\frac{11}{185}\right)^2 + \left(\frac{8}{185}\right)^2} = \frac{\sqrt{185}}{185}$

$\tan \phi = \frac{8}{185} \cdot \frac{185}{11} = \frac{8}{11}$ $\quad \phi = 36^\circ$

$\frac{11}{185} - \frac{8}{185}i = \frac{\sqrt{185}}{185} e^{(36^\circ)i} = K$

$y_c = 3\left(\frac{\sqrt{185}}{185} e^{i(36^\circ + 2t)}\right)$ The real part of this:

$$\boxed{y_p = \frac{3\sqrt{185}}{185}\cos(2t + 36^\circ)}$$

(9) $y'' + 4y' + 20y = -3\sin 2t$

$\lambda^2 + 4\lambda + 20 = 0$

$$\frac{-4 \pm \sqrt{16-80}}{2} = \frac{-4 \pm \sqrt{-64}}{2} = \frac{-4 \pm 8i}{2} = -2 \pm 4$$

$y_h = c_1 e^{-2t}\cos 4t + c_2 e^{-2t}\sin 4t$

$y'' + 4y' + 20y = e^{2ti}$ try $y = ke^{2ti}$

$-2ke^{2it} + 8ike^{2it} + 20ke^{2it} = e^{2it}$

$k(-2 + 8i + 20)$ $\quad k = \frac{1}{18+8i} \cdot \frac{18-8i}{18-8i} = \frac{18-8i}{324+64} = \frac{1}{3}$

$y_c = \left(\frac{18}{388} - \frac{8}{388}i\right)e^{it}$ write y_c in polar form

$r = \sqrt{\left(\frac{18}{388}\right)^2 + \left(\frac{8}{388}\right)^2} = \frac{\sqrt{388}}{388}$

$\tan\varphi = \frac{-8}{388} \cdot \frac{388}{18} = \frac{-8}{18} = \frac{-4}{9}$ $\quad \varphi = -24'$

$\frac{18}{388} - \frac{8}{388}i = \frac{\sqrt{388}}{388}e^{(-24°)i}$

$y_c = \left(\frac{18}{388} - \frac{8}{388}i\right)e^{it} = \frac{\sqrt{388}}{388}e^{(-24)i}e^{it}$

$$\boxed{y_p = \frac{-3\sqrt{388}}{388}\sin(2t - 24°)}$$