Homework problems on modeling with PDE's

(Some parts of these problems were already done in the problem set dealing with transport equations.)

Suppose that $c(x,t)$ denotes the density of a bacterial population.

Here "x" may be a scalar, or it may be a vector (x_1, x_2, x_3) in 3-space, depending on the description.

For each of the following descriptions, you must provide a differential equation that provides a model of the situation.

You are not asked to *solve* any equations. The *only* point of the exercise is to get you used to "translate" from word descriptions to equations.

The values of constants that are used, for velocities, diffusion coefficients, and so on, are arbitrary and have no physical meaning.

A. These first problems all assume dimension 1. We think of the bacteria as living inside a tube of infinite length (perhaps a very long ventilation system). The density is assumed to be uniform in each cross-section, so we model by $c = c(x,t)$ with $-\infty < x < +\infty$.

1. Bacteria are transported by an air current blowing "east" (towards $x > 0$) at 5 m/s, and they grow exponentially with a doubling time of 1 hour.
2. Bacteria are transported by an air current blowing "west" at 5 m/s, and they grow exponentially with a doubling time of 1 hour.
3. Bacteria are transported by an air current blowing east at 5 m/s, and they grow exponentially with a doubling time of 1 hour for small populations, but nutrients are restricted, so the density can never be more than 100m^{-1}.
4. Bacteria are transported by an air current blowing east at 5 m/s, and they grow exponentially with a doubling time of 1 hour, and they also move randomly with a diffusion coefficient of 10^{-3}.
5. Suppose now that there is a source of food at $x = 0$, and bacteria area attracted to this source. Model the potential as $V(x) = e^{-x^2}$.

B. Now we work in dimension 3. Bacteria are moving in space.

1. Bacteria are transported by an air current blowing parallel to the vector $v = (1, -1, 2)$ at 5 m/s, and they grow exponentially with a doubling time of 1 hour.
2. Bacteria are transported by an air current blowing parallel to the vector $v = (1, -1, 2)$ at 5 m/s, and they grow exponentially with a doubling time of 1 hour for small populations, but nutrients are restricted, so the density can never be more than 100 (in appropriate units).
3. Bacteria are transported by an air current blowing parallel to the vector $v = (1, -1, 2)$ at 5 m/s, and they also move randomly with a diffusion coefficient of 10^{-3}.