

A1. Transport velocity is 5 m/s.

Doubling time is 1 hr = 3600 sec, so $2 = e^{\lambda 3600}$ means $\lambda = \frac{\ln 2}{3600}$. So rate of reproduction is $\sigma = \lambda c = \frac{\ln 2}{3600} c$ and equation is:

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + \frac{\ln 2}{3600} c.$$

A2. Transport velocity is now -5 m/2, and λ is as in A1.

$$\frac{\partial c}{\partial t} = 5 \frac{\partial c}{\partial x} + \frac{\ln 2}{3600} c.$$

A3. The reaction term is now different, since the rate of reproduction must be negative when $c > 100$. The simplest formula is $\lambda c(1 - c/100)$, where $\lambda = \frac{\ln 2}{3600}$. Then an equation is:

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + \frac{\ln 2}{3600} c(1 - c/100).$$

A4. Just add diffusion to A1:

$$\frac{\partial c}{\partial t} = 10^{-3} \frac{\partial^2 c}{\partial x^2} - 5 \frac{\partial c}{\partial x} + \frac{\ln 2}{3600} c.$$

A5. Since $V(x) = e^{-x^2}$, we have that $V'(x) = -2xe^{-x^2}$ and $V''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$. Therefore, the chemotactic term is (with proportionality constant $\alpha = 1$): $-\frac{\partial c}{\partial x} V' - cV'' = 2xe^{-x^2} \frac{\partial c}{\partial x} + 2e^{-x^2} c - 4x^2e^{-x^2} c$, and therefore, putting it all together:

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + \frac{\ln 2}{3600} c(1 - c/100) + 2xe^{-x^2} \frac{\partial c}{\partial x} + 2e^{-x^2} c - 4x^2e^{-x^2} c$$

B1. Transport: velocity is $v = 5(1, -1, 2) = (5, -5, 10)$, and we compute:¹

$$\operatorname{div}((5, -5, 10)c) = \operatorname{div}(5c, -5c, 10c) = 5 \frac{\partial c}{\partial x} - 5 \frac{\partial c}{\partial y} + 10 \frac{\partial c}{\partial z}$$

so using σ as earlier, we end up with:

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + 5 \frac{\partial c}{\partial y} - 10 \frac{\partial c}{\partial z} + \frac{\ln 2}{3600} c.$$

B2.

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + 5 \frac{\partial c}{\partial y} - 10 \frac{\partial c}{\partial z} + \frac{\ln 2}{3600} c(1 - c/100).$$

B3.

$$\frac{\partial c}{\partial t} = -5 \frac{\partial c}{\partial x} + 5 \frac{\partial c}{\partial y} - 10 \frac{\partial c}{\partial z} + 10^{-3} \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right).$$

¹As a student pointed out, there is an error in this answer. If the velocity is $(5, -5, 10)$, the speed would be $\sqrt{5^2 + 5^2 + 10^2} = \sqrt{150}$, not 5. So there is a scalar factor missing.