Prob. 25 in chap. 4

Part (e) is ambiguously stated, but let us see what we can say.

Let us first look for the general solution of

$$N' = (b - a \ln N)N, \tag{1}$$

where a and b are two constants. If we introduce a new variable $z = \ln N$, then

$$z' = \frac{N'}{N} = b - az$$

and therefore $z(t) = e^{-at}z(0) + (1 - e^{-at})\frac{b}{a}$, or:

$$\ln N(t) = e^{-at} \ln N(0) + (1 - e^{-at})\frac{b}{a}\,.$$

On the other hand, the general solution of

$$N' = ke^{-at}N \tag{2}$$

can be obtained by a similar trick: let $z = \ln N$, then

$$z' = \frac{N'}{N} = ke^{-at},$$

so

$$z(t) = z(0) - \frac{k}{a}e^{-at},$$

i.e.:

$$\ln N(t) = \ln N(0) - \frac{k}{a}e^{-at},$$

In conclusion, both for (1) and (2) the solutions have the forms

$$\alpha_1 + \alpha_2 e^{-at}\,.$$

But that is about all that one can say.